## Your Name

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Page	Total Points	Score
2	22	
3	10	
4	20	
5	16	
6	15	
7	12	
8	22	
9	20	
10	13	
Total	150	

- You will have 2.5 hours to complete the test.
- This test is closed notes and closed book and you may not use a calculator.
- In order to receive full credit, you must **show your work**. Please write out your computations on the exam paper.
- PLACE A BOX AROUND YOUR FINAL ANSWER to each question where appropriate.

1. (22 pts) Find the following limits. If a limit does not exist, explain why and give  $\pm \infty$  when appropriate.

(a) (3 pts) 
$$\lim_{x \to 5} \frac{2x^2 - 50}{x^2 - 3x - 10}$$
 (b) (3 pts)  $\lim_{x \to 0} \frac{x}{\tan^{-1}(4x)}$ 

(c) (3 pts) 
$$\lim_{x \to -\infty} \frac{\sqrt{x^2 + 4x}}{4x + 1}$$
 (d) (3 pts)  $\lim_{x \to 5^+} \frac{2 - x^2}{x - 5}$ 

(e) (5 pts) 
$$\lim_{x \to 2} \frac{x - \sqrt{3x - 2}}{x^2 - 4}$$
 (f) (5 pts)  $\lim_{x \to \infty} (e^x + x)^{1/x}$ 

(e) (5 pts) 
$$\lim_{x \to 2} \frac{x - \sqrt{3x - 2}}{x^2 - 4}$$

2. (6 pts) Sketch the graph of  $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ x^2 - 3 & \text{if } 0 \le x \le 3. \end{cases}$  Then, find the numbers at which  $f(x) \\ 2x - 5 & \text{if } x > 3 \end{cases}$  is discontinuous. Specify whether f is continuous from the right, from the left, or neither at any

discontinuities.



3. (4 pts) Use the definition of the derivative to find f'(x) if  $f(x) = 7 + 5x - 3x^2$ .

4. (10 pts) Find and simplify the derivatives of the following functions.

a) 
$$y = t^2 \arctan(2t)$$
 b)  $f(x) = \frac{e^{1/x}}{x^2}$ 

5. (10 pts) Find and simplify the derivatives of the following functions.

a) 
$$y = \ln\left(\frac{\sqrt{x}}{x^2 + 1}\right)$$
 b)  $g(x) = \tan^2(\sin 5x)$ 

- 6. (9 pts) Find the derivatives of the following functions.
  - (c) (5 pts)  $y = (\cos x)^{2x}$

(d) (4 pts) 
$$F(x) = \int_{x^5}^1 \frac{t^2}{\sqrt{1-t^4}} dt$$

7. (7 pts)

(a) (5 pts) Given 
$$x^2 - xy + y^2 = 1$$
 find  $\frac{dy}{dx}$ .

(b) (2 pts) Find an equation of the tangent line to  $x^2 - 2xy + y^2 = 1$  at (1, 0)

8. (5 pts) A ladder 10 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when it's base is 3 feet from the wall?

9. (4 pts) Find the absolute maximum and absolute minimum of  $f(x) = x^3 - 6x^2 + 9x + 3$  on [-1, 2]

10. (6 pts) An open-top box is to be made whose base length is twice its width. If 600 square centimeters is to be used in its construction, what dimensions maximize the volume of the box?

## 11. (12 pts) Let $f(x) = xe^{-x}$

(a) (3 pts) Find the intervals of increase/decrease.

- (b) (2 point) Find the local extrema. Classify as a maxima or minima.
- (c) (3 point) Find the intervals of concavity.

- (d) (2 point) Find any inflection points.
- (e) (2 pt) Find the horizontal asymptotes, if any.

- 12. (6 pts) Given  $f(x) = x^2 + 1$ , estimate the area under the curve from [0, 8] with n = 4 using:
  - a) Left endpoints. b) Midpoints.

13. (8 pts) Find the following indefinite integrals.

a) 
$$\int \left(x - \frac{1}{x}\right)^2 dx$$
 b)  $\int \sec x (\sec x + \tan x) dx$ 

14. (8 pts) Find the following indefinite integrals.

a) 
$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$
 b)  $\int \frac{x^3}{1+x^4} dx$ 

15. (10 pts) Find the following indefinite integrals.

a) 
$$\int \frac{x}{(x-3)^2} dx$$
 b)  $\int \frac{x+4}{x^2+1} dx$ 

16. (10 pts) Find the following definite integrals.

a) 
$$\int_1^9 \frac{\sqrt{u} - 2u^2}{u} du$$

b) 
$$\int_0^{\pi/4} \sec^2 x \tan^2 x dx$$

17. (5 pts) A particle is moving with the given data. Find the position of the particle.  $a(t) = 2t + \sin t + 1$ , s(0) = 1, v(0) = -2

- 18. A particle moves along the *x*-axis with velocity function  $v(t) = t^2 4$ , where *v* is measured in meters per second.
  - (a) (3 pts) Find the displacement over the time interval [0,3]

(b) (5 pts) Find the distance traveled over the time interval [0,3].