Your Name (Printed)

End Time

Directions

- You will have **one** hour to complete the test. No extra time will be given, use your time wisely!
- This test is closed notes and closed book and you may **not** use a calculator.
- In order to receive full credit, you must **show your work** using correction notation. Please write out your computations on the exam paper. All answers should be simplified with the correct units where necessary.
- Simplify all answers by finding a common denominator, factoring out greatest common factors and canceling, when appropriate. In the exam you will be instructed to do this when the directions ask you to **simplify**.
- Solutions must be clearly identified by **placing a box around** $\boxed{\text{your final answer}}$ to each question, when appropriate.

Total Points	Your Score	Percent
125		

8:42

1. (10 points) Differentiate the following functions. Do not simplify your answers.

(a) $y = 4x^5 + 8\sqrt[4]{x^5} + \sin 5$

$y = 4x^5 + 8x^{5/4} + \sin 5$

$\frac{dy}{dx} = 20x^4 + \frac{40}{4}x^{1/4} + 0$

$\boxed{\frac{dy}{dx} = 20x^4 + 10\sqrt[4]{x}}$

(b) $f(x) = \csc(2x) - \cot\left(\frac{x}{2}\right)$

$f'(x) = -\csc(2x)\cot(2x)\cdot 2 + \frac{1}{2}\csc^2(x/2)$

$\boxed{f'(x) = -2\csc(2x)\cot(2x) + \frac{1}{2}\csc^2(x/2)}$

2. (10 points) Find the derivatives of the following functions. Simplify.

(a) $y = 2^{x \ln x}$

$y' = (\ln 2)\, 2^{x\ln x}\left(1\ln x + x\cdot\frac{1}{x}\right)$

$\boxed{y' = (\ln 2)\, 2^{x\ln x}(\ln x + 1)}$

(b) $h(x) = \frac{e^{1/x}}{x^2}$

$h'(x) = \frac{x^2 e^{1/x}(-1x^{-2}) - e^{1/x}\cdot 2x}{x^4}$

$= \frac{-e^{1/x} - 2xe^{1/x}}{x^4}$

$\boxed{h'(x) = \frac{-e^{1/x}(1+2x)}{x^4}}$

3. (20 points) Find the derivatives of the following functions. **Simplify.**

(a) $y = \dfrac{x^2 - 4x + 2}{\sqrt{x}}$

$y = x^{3/2} - 4x^{1/2} + 2x^{-1/2}$

$y' = \frac{3}{2} x^{1/2} - 2x^{-1/2} - 1x^{-3/2}$

$y' = \frac{3x^{1/2}}{2}\frac{x^{3/2}}{x^{3/2}} \quad \frac{2}{x^{1/2}}\frac{2x}{2x} - \frac{1}{x^{3/2}}\frac{2}{2}$

$$\boxed{y' = \frac{3x^2 - 4x - 2}{2x^{3/2}}}$$

(b) $y = \log_{10}(\sin(3x))$

$y' = \dfrac{1}{(\ln 10)\sin(3x)} \cdot 3 \cdot \cos(3x)$

$$\boxed{y' = \frac{3\cos(3x)}{\ln 10 \sin 3x}}$$

$$\boxed{y' = \frac{3\cot 3x}{\ln 10}}$$

(c) $f(x) = \tan\left(\dfrac{x}{1+x^2}\right)$

$f'(x) = \sec^2\left(\frac{x}{1+x^2}\right)\left(\frac{(1+x^2) - x \cdot 2x}{(1+x^2)^2}\right)$

$$= \boxed{\sec^2\left(\frac{x}{1+x^2}\right)\left(\frac{1-x^2}{(1+x^2)^2}\right)}$$

(d) $y = \arctan(e^{7x^2})$

$y' = \dfrac{1}{1+(e^{7x^2})^2} \cdot e^{7x^2} \cdot 14x$

$$\boxed{y' = \frac{14x\, e^{7x^2}}{1 + e^{14x^2}}}$$

5. (15 points) Differentiate the following functions. Simplify.

(a) $y = x\arcsin(2x) + \frac{1}{2}\sqrt{1-4x^2}$

$$y' = 1\arcsin(2x) + x \cdot \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 + \frac{1}{2}(1-4x^2)^{-1/2}(-8x)$$

$$= \arcsin(2x) + \frac{2x}{\sqrt{1-4x^2}} - \frac{2x}{\sqrt{1-4x^2}}$$

$$= \boxed{\arcsin(2x)}$$

(b) $f(x) = \ln\left(\frac{(x^2-4)^5}{\sqrt{2x+5}}\right)$

$$= 5\ln(x^2-4) - \frac{1}{2}\ln(2x+5)$$

$$f'(x) = \frac{5}{x^2-4} \cdot 2x - \frac{1}{2}\frac{1}{(2x+5)}2$$

$$= \frac{10x}{(x^2-4)}\frac{(2x+5)}{(2x+5)} - \frac{1}{2x+5}\frac{(x^2-4)}{(x^2-4)}$$

$$= \frac{20x^2 + 50x - x^2 + 4}{(x^2-4)(2x+5)}$$

$$= \boxed{\frac{19x^2 + 50x + 4}{(x^2-4)(2x+5)}}$$

(c) $y = (\cos x)^x$

$$\ln y = \ln(\cos x)^x$$

$$\ln y = x\ln(\cos x)$$

$$\frac{1}{y}\frac{dy}{dx} = 1\ln(\cos x) + x \cdot \frac{1}{\cos x}(-\sin x)$$

$$\frac{dy}{dx} = (\ln(\cos x) - x\tan x)\,y$$

$$\boxed{\frac{dy}{dx} = (\ln(\cos x) - x\tan x)(\cos x)^x}$$

6. (6 points) Find the equation of the tangent line to $f(x) =$ [redacted] at the point $(0, 1)$. Give your answer in slope-intercept form.

$= \sqrt{1+4\sin x}$

$$f'(x) = \frac{1}{2}(1+4\sin x)^{-1/2} \cdot 4\cos x$$

$$= \frac{2\cos x}{\sqrt{1+4\sin x}}$$

$$m = \frac{2\cos 0}{\sqrt{1+4\sin 0}} = 2$$

$$y - 1 = 2(x - 0)$$

$$\boxed{y = 2x + 1}$$

7. (6 points) Find all values of x for which the graph of $f(x) = \sin x + \cos^2 x$ has a horizontal tangent line. Note, you only have to find the x-coordinate.

$$f'(x) = \cos x + 2\cos x(-\sin x)$$

$$0 = \cos x\,(1 - 2\sin x)$$

$0 = \cos x$ $\qquad$ $0 = 1 - 2\sin x$

$1/2 = \sin x$

$\boxed{x = \pi/2 + \pi n}$ $\qquad$ $\boxed{x = \pi/6 + 2\pi n}$

$\boxed{x = 5\pi/6 + 2\pi n}$

8. (5 points) Find the 46th derivative of $g(x) = e^{5x}$.

$$g'(x) = 5e^{5x}$$

$$g''(x) = 5^2 e^{5x}$$

$$\boxed{g^{(46)}(x) = 5^{46} e^{5x}}$$

9. (6 points) Given $xe^y = y - 1$ find $\frac{dy}{dx}$.

$$1e^y + xe^y \, dy/dx = dy/dx - 0$$

$$xe^y \frac{dy}{dx} - \frac{dy}{dx} = -e^y$$

$$\frac{dy}{dx}(xe^y - 1) = -e^y$$

$$\boxed{\frac{dy}{dx} = \frac{-e^y}{xe^y - 1}} \quad \text{OR} \quad \boxed{\frac{e^y}{1 - xe^y}}$$

10. (6 points) Find the equation of the tangent line to $x^2 + 4xy + y^2 = 13$ at the point $(2, 1)$. Give your answer in slope-intercept form.

$$2x + 4y + 4x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$(4x + 2y) \, dy/dx = -2x - 4y$$

$$\frac{dy}{dx} = \frac{-2(x+2y)}{2(2x+y)} = \frac{-(x+2y)}{(2x+y)}$$

$$m = -\frac{(2+2)}{4+1} = -4/5$$

$$y - 1 = -4/5(x - 2)$$

$$y - 1 = -4/5x + 8/5$$

$$\boxed{y = -4/5x + 13/5}$$

11. (7 points) Each side of a square is increasing at a rate of 6 cm/sec. At what rate is the area of the square increasing when the area of the square is 16 cm^2? Give your answer with proper units.

$dx/dt = 6$ cm/sec
want dA/dt
when $A = 16$

$16 = x^2$

$x = 4$

$$A = x^2$$

$$\frac{dA}{dt} = 2x \frac{dx}{dt}$$

$$\frac{dA}{dt} = 2(4)(6)$$

$$\boxed{\frac{dA}{dt} = 48 \text{ cm}^2/\text{sec}}$$

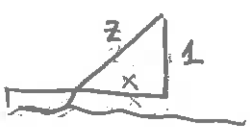
12. (7 points) A boat is pulled into a dock by a rope attached to the bow (front) of the boat and passing through a pulley on the dock that is 1 m higher than the boat. If the rope is pulled in at a rate of 2 m/sec, how fast is the boat approaching the dock when it is 3 m from the dock? Give your answer with proper units.

$dz/dt = 2$ m/s
want dx/dt
when $x = 3$

$z^2 = 3^2 + 1^2$

$z^2 = 10$

$z = \sqrt{10}$



$$1^2 + x^2 = z^2$$

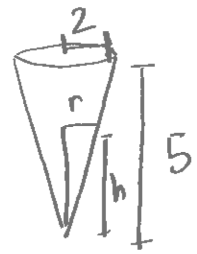
$$2x\, dx/dt = 2z\, dz/dt$$

$$x\, dx/dt = z\, dz/dt$$

$$3(dx/dt) = \sqrt{10}\,(2)$$

$$\boxed{\frac{dx}{dt} = \frac{2\sqrt{10}}{3} \text{ m/sec}}$$

13. (7 points) A water tank has the shape of a cone (point down) with height 5 m and base radius 2 m. Suppose the tank is full initially, and water is leaking from the tank at a rate of $0.5\ \text{m}^3/\text{min}$. How fast is the water level falling when the water is 3 m deep? $V = \frac{\pi}{3} r^2 h$



$\frac{r}{h} = \frac{2}{5}$

$r = \frac{2}{5} h$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3} \left(\frac{2}{5} h\right)^2 h$

$V = \frac{4\pi}{25} h^2 \frac{dh}{dt}$

$-\frac{1}{2} = \frac{4\pi}{25} \cdot 3^2 \frac{dh}{dt}$

$-\frac{1}{2} = \frac{4\pi}{25} \cdot 9 \frac{dh}{dt}$

$-\frac{1}{2} = \frac{36\pi}{25} \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-25}{72\pi} \text{m}/\text{min}}$ $\boxed{\frac{-25}{72\pi}}$

$\frac{dV}{dt} = -\frac{1}{2}\ \text{m}^3/\text{min}$

want $\frac{dh}{dt}$ when $h = 3$

14. (6 points) Find the linearization of $f(x) = e^x$ at $a = 0$ and use it to estimate $e^{0.1}$

$f'(x) = e^x$ $a = 0$, $f(0) = 1$

$m = f'(0) = 1$

$y - 1 = 1(x - 0)$

$\boxed{y = x + 1}$

$e^{0.1} \approx 0.1 + 1$

$= \boxed{1.1}$

15. (8 points) A particle moves along a horizontal line so that its coordinate at time t is $x = t^3 - 3t + 5, t \geq 0$.

(a) (2 points) Find the velocity and acceleration functions.

$v(t) = 3t^2 - 3$

$a(t) = 6t$

(b) (2 points) When is the particle at rest?

$0 = 3(t^2 - 1)$

$0 = 3(t-1)(t+1)$

$\boxed{t = 1}$

(c) (4 points) Draw a diagram of the particle's position and find the distance the particle travels in first three seconds.

① $t = 0 \quad x = 5$

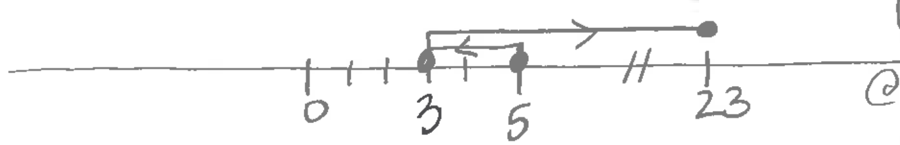
② $t = 1 \quad x = 1 - 3 + 5$

$= 3$

$@ t = 3$

$x = 27 - 9 + 5$

$x = 23$



total dist: $2 + 20 = \boxed{22}$

16. (6 points) Use differentials to estimate the amount of paint needed to apply a coat of paint 0.1 **cm** thick to a hemispherical dome with radius 10 **m**. Give your final answer with proper units. (The volume of a sphere is $V = \frac{4}{3}\pi r^3$) hemisphere — half

$V = \frac{2}{3}\pi r^3$

$dV = 2\pi r^2 dr$

$dv = 2\pi (10^2)(0.001)$

$= 200\pi (0.001)$

$= \boxed{0.2\pi \ m^3}$

0.1 cm $= 0.1 \div 100$ m

$= 0.001$ m

OR 10 m $= 10 * 100$ cm

$= 1000$ cm

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