

Your Name

Your Signature

Instructor Name

Problem	Total Points	Score
1	8	
2	14	
3	12	
4	10	
5	16	
6	14	
7	12	
8	14	
Extra Credit	(5)	
Total	100	

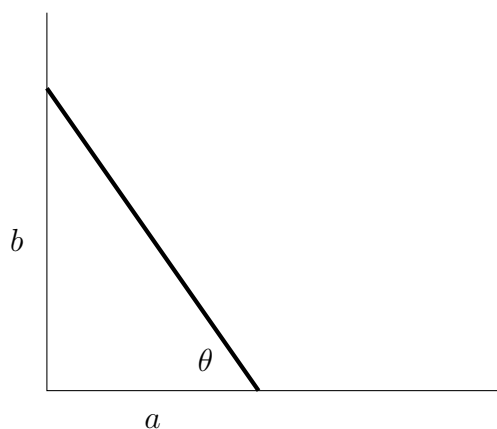
- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work**. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- Raise your hand if you have a question.

1. (8 points)

(a) Find the linearization of  $f(x) = \sqrt[3]{x}$  at  $a = 8$ .

(b) Use your result in part (a) to approximate  $\sqrt[3]{9}$ .

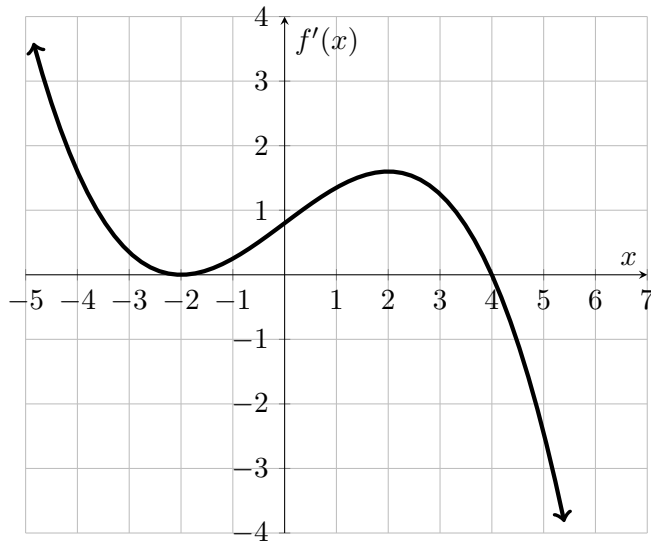
2. (14 points) A ladder 12 ft long is propped against a wall. At the moment the ladder makes a  $60^\circ$  angle to the ground, the bottom of the ladder is moving at a rate of 1 ft/sec away from the wall.



Find the rate at which the top of the ladder is sliding down the wall at this same moment. Include units in your answer.

3. (12 points)

The graph of the *derivative*  $f'$  of a function  $f$  is shown.



(a) Determine the critical points of  $f(x)$ .

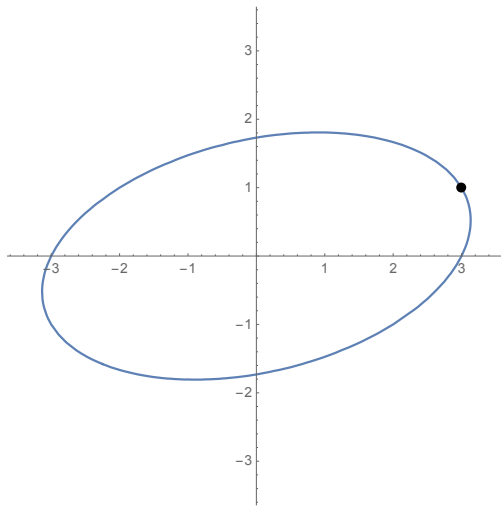
(b) On what intervals is  $f$  increasing or decreasing? Use interval notation.

(c) At what values of  $x$  does  $f$  have a local maximum or minimum?

(d) On what intervals is  $f$  concave up or concave down? Use interval notation.

4. (10 points) Consider the curve given by the implicitly defined function

$$x^2 - xy + 3y^2 = 9$$



- (a) On the figure above, sketch the tangent line to the curve at the point  $(3, 1)$ .
- (b) Find the equation of the tangent line to the curve at the point  $(3, 1)$  (shown on the picture by the solid dot).

5. (16 points) We want to sketch a graph of a function  $f(x)$  with certain specified properties.

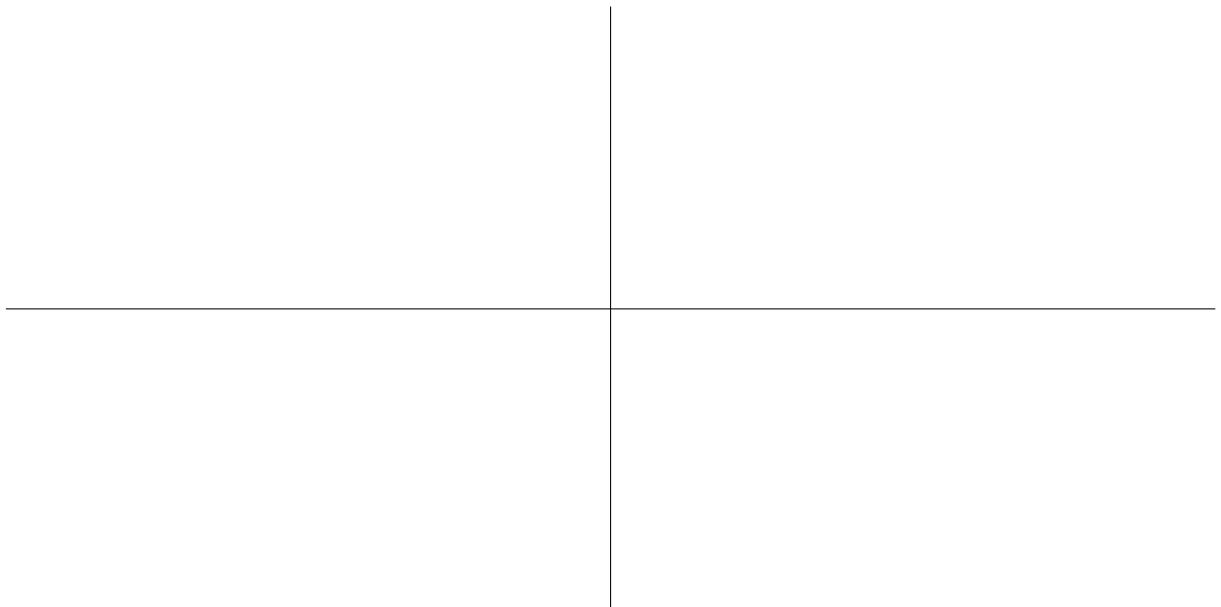
(a) Fill in the following tables. (You can use words or pictures.)

function information	what you conclude about the behavior of $f$
$\lim_{x \rightarrow 4} f(x) = -\infty$	
$\lim_{x \rightarrow -\infty} f(x) = 0$	
$\lim_{x \rightarrow \infty} f(x) = \infty$	
$f(0) = 1$	
$f'(-3) = 0$	

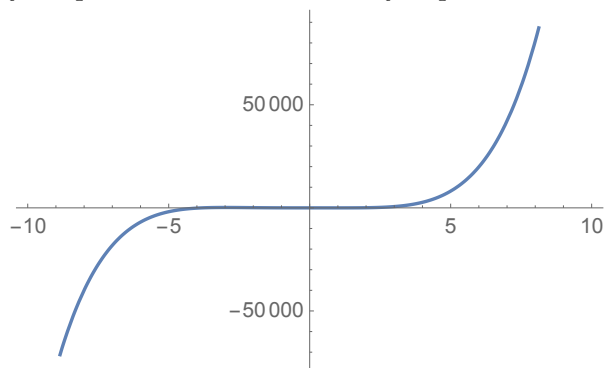
$x$	$-\infty < x < -3$	$-3 < x < 4$	$4 < x < \infty$
sign of $f'(x)$	+	-	+
Behavior of $f(x)$			

$x$	$-\infty < x < -5$	$-5 < x < 4$	$4 < x < \infty$
sign of $f''(x)$	+	-	-
Behavior of $f(x)$			

(b) Sketch the graph of  $f$  that has all of the properties listed in the tables. Label **on the graph** any local maxima and minima, any inflection points, any horizontal asymptotes and vertical asymptotes (draw with dashed lines, label with their equations), and any roots of the function that were specified in the table. **Label the axes (add tick marks) to indicate important  $x$ - and  $y$ -values.** Your sketch does not need to be to scale.



6. (14 points) Consider the function  $g(x) = 2x^5 + 5x^4 - 10x^3 + 2$ .
- (a) Determine all critical values of  $g(x)$  and identify each value as the location of a local maximum, a local minimum, or neither. Justify your answer.
- (b) Consider  $g(x)$  on the interval  $[-1, 2]$ . Determine the absolute maximum and minimum for  $g(x)$  on that interval. (Hint:  $g(2) = 66$ )
- (c) Below is a computer-generated plot of  $g(x)$  on the intervals  $[-10, 10]$ . Does this plot contradict your previous answers? Briefly explain.



7. (12 points) Evaluate the limits below. Indicate a use of L'Hôpital's Rule by a symbol (such as **L'H** or **H**) over the equal sign.

(a)  $\lim_{t \rightarrow 4} \frac{\sin(\pi t)}{t^2 - 16}$

(b)  $\lim_{x \rightarrow 0} \frac{x - 4 \cos(\pi x)}{3e^x}$

(c)  $\lim_{x \rightarrow \infty} x(5 - 5e^{\frac{1}{x}})$



8. (14 points) A box with a square base must have a volume of  $2500 \text{ in}^3$ . Wood for the base and sides costs \$0.50 per square inch and wood for the decorative top costs \$2 per square inch. What are the dimensions of the least expensive box?

(a) Draw and label a picture.

(b) Explicitly state what it is you must minimize or maximize. (One or two words is sufficient.)

(c) Write the quantity in part (b) as a function of one variable and state its domain.

(d) Answer the question and justify your answer.

**Extra Credit:** (5 points) Is the statement below TRUE or FALSE? Justify your answer.

Suppose  $t > 0$ . An antiderivative of

$$f(t) = \arcsin\left(\frac{1}{t}\right) - \frac{1}{\sqrt{t^2 - 1}}$$

is

$$F(t) = t \arcsin\left(\frac{1}{t}\right) - 4.$$