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Honor code:

If not using a printer, just sign your name. No need to write this all out.

- I will not use any online resource or electric means of any kind*.
- I have no prior knowledge of the problems on this exam.
- I am taking this exam without the aid of anyone or anything.

signature: ______________________

*: Exceptions: (1) Use of tablet to complete exam; (2) Use of devices to download/scan/upload exam.

- The instructor reserves the right to follow up live to discuss certain aspects of this exam for verification.
- A 10 point penalty can be enforced if the stated honor code is determined to be violated at the sole discretion of the instructor.
- Failure to respond/meet with instructor will result in the automatic application of a 20 point penalty.

Instructions:

- You have **70 minutes** to print/complete exam OR complete exam digitally on a tablet OR complete exam on separate paper.. ***Failure to sign the honor code, not being on Zoom, not inputting your UA ID, or not finishing/stopping within 70 minutes will result in a penalty***.
- When you finished, or 70 minutes after you download the exam (whichever comes first), slowly move each page in front of the camera. At this point, you cannot make any changes, and you can then scan/submit the exam to Blackboard. Do not share the exam with anyone.
- No notes. No book. No calculator. No friends.
- In order to receive full credit, you must **show your work.** Simplify obvious expressions.
- Read the directions carefully. The form of the answer or solution method may be specified in some places.
- **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.

1 (14 points) Evaluate the following limits. Justify your answers with words and/or any relevant algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim\limits_{x \to -4} \dfrac{x^2 - 16}{x^2 + x - 12}$

$$= \lim_{x \to -4} \frac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}(x-3)}$$

$$= \lim_{x \to -4} \frac{x-4}{x-3} = \frac{-8}{-7} = \boxed{8/7}$$

(b) $\lim\limits_{t \to -5} \dfrac{2 + 4t}{t^2 + 2} = \dfrac{2 + 4(-5)}{(-5)^2 + 2}$

$$= \frac{2-20}{27} = \frac{-18}{27} = \boxed{-2/3}$$

(c) (i) $\lim\limits_{x \to -1^-} \sqrt{x^2 - 1} = \sqrt{(-1)^2 - 1} = \sqrt{1-1} = 0$

as x ~~goes~~ goes to
-1 from the left,
$x^2 - 1 > 0$

(ii) Why do we not evaluate $\lim\limits_{x \to -1^+} \sqrt{x^2 - 1}$? Explain using a sentence.

As x goes to -1 from the right, $x^2 - 1 < 0$,
and we can't take square root of negative number.

2 (10 points) Evaluate the following limits. Justify your answers with words and/ or any relevant algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim_{x\to 1} \frac{x-1}{\sqrt{2x-1}+1}$ $= \frac{1-1}{\sqrt{1}+1} = \boxed{0}$

or conjugate:

$$\lim_{x\to 1} \frac{(x-1)(\sqrt{2x-1}-1)}{(\sqrt{2x-1}+1)(\sqrt{2x-1}-1)}$$

$$= \lim_{x\to 1} \frac{(x-1)(\sqrt{2x-1}-1)}{2x-1-1}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{2x-1}-1)}{2\cancel{(x-1)}} = \frac{\sqrt{1}-1}{2} = \boxed{0}$$

(b) $\lim_{x\to\infty} \cos\left(\frac{\pi x^2 - 5x}{6x^2+2x-9}\right)$

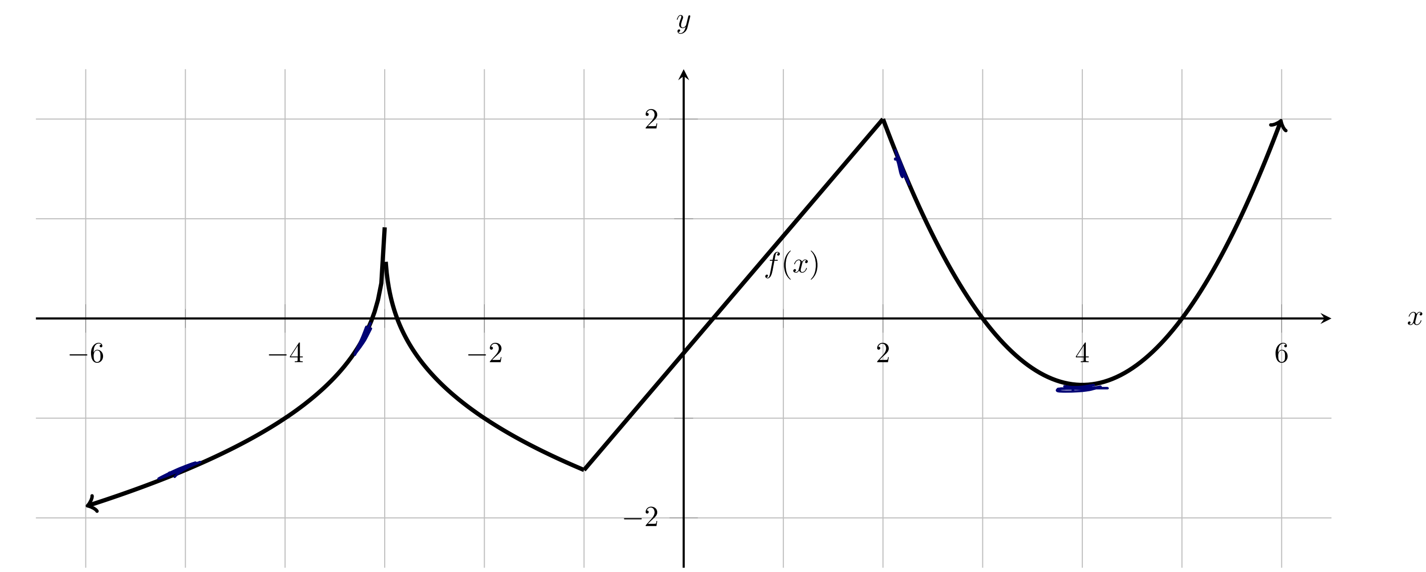
$$= \cos\left(\lim_{x\to\infty} \frac{\pi x^2 - 5x}{6x^2+2x-9} \cdot \frac{1/x^2}{1/x^2}\right)$$

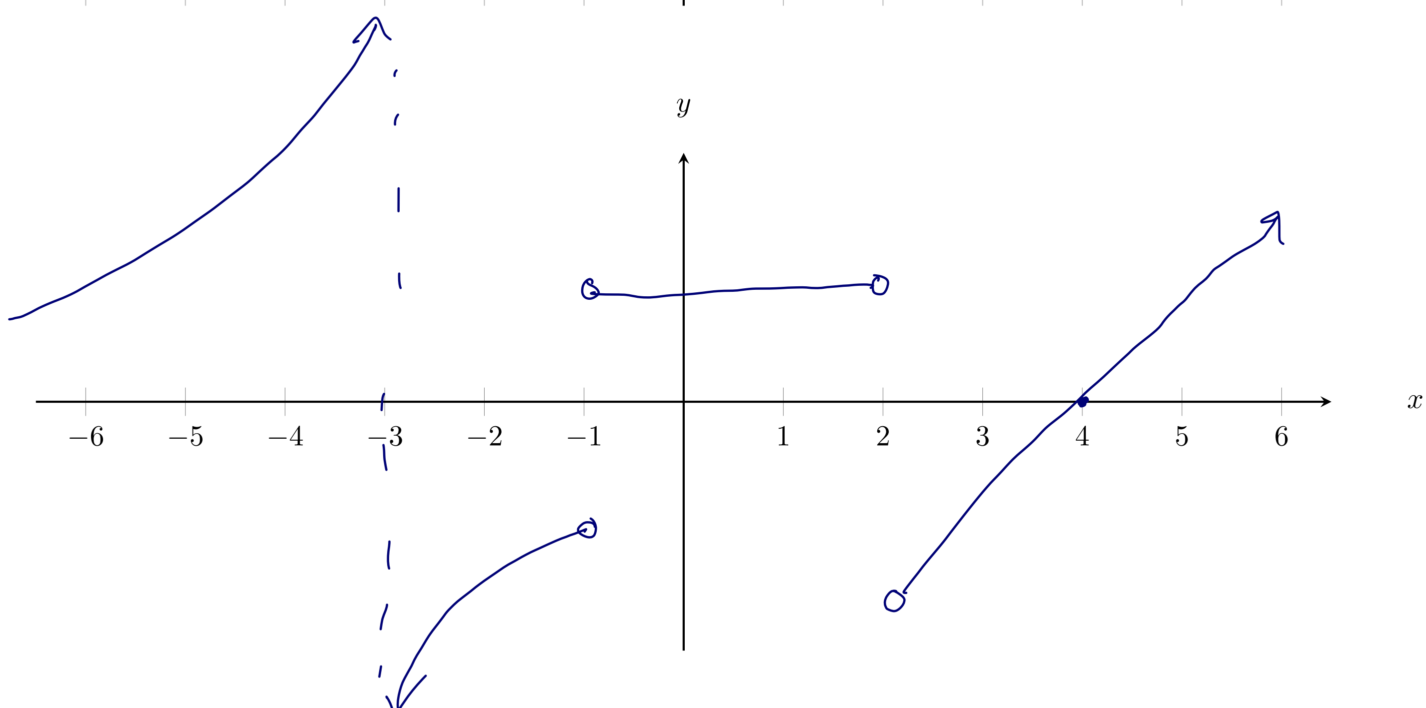
$$= \cos\left(\lim_{x\to\infty} \frac{\pi - \cancel{5/x}^{\,0}}{6 + \cancel{2/x}^{\,0} - \cancel{9/x^2}^{\,0}}\right)$$

$$= \cos(\pi/6) = \boxed{\sqrt{3}/2}$$

$\boxed{3}$ (14 points)

(a) The graph of $f(x)$ is shown on the top set of axes. Sketch the graph of $f'(x)$ on the second set of axes.





(b) What is the domain of $f'(x)$? Write your answer using interval notation.

Domain: $(-\infty,-3)\cup(-3,-1)\cup(-1,2)\cup(2,\infty)$

4 (10 points) In the first few years after a coal mine's operation, the total deposit of coal (in millions of tons) t years after opening is approximately

$$C(t) = 300 - \frac{t^{3/2}}{2}.$$

(a) Find the average rate of change of the amount of coal in the deposit from the opening of the mine to year 4. Include correct units in your answer.

$$\text{Ave. rate} = \frac{C(4)-C(0)}{4-0} = \frac{\cancel{300} - \frac{4^{3/2}}{2} - (\cancel{300}-0)}{4}$$

$$= -\frac{(\sqrt{4})^3}{8}$$

$$= \frac{-2^3}{8} = \boxed{-1 \ \frac{\text{million tons}}{\text{year}}}$$

(b) It is a fact that $C'(t) = -\frac{3}{4}\sqrt{t}$. Compute $C'(4)$ and indicate what this quantity tells us about the mine. Write your answer in a sentence. Again, include correct units in your description.

$$C'(4) = -\frac{3}{4}\sqrt{4} = -\frac{3}{4}(2) = -3/2$$

$C'(4) = -3/2$ million tons/year.

At the 4 year mark, the change in the amount of coal in the deposit is -1.5 million tons per year.

5 (14 points) Let $g(x) = \frac{2x^2+6x}{x^2-9}$. $= \frac{2(x+3)}{(x+3)(x-3)}$

(a) What is the domain of g? Write your answer using interval notation.

Domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

(b) Use limits to determine all **vertical** asymptotes of $g(x)$. Show your work clearly and justify your conclusion using limits. Write the equations of the vertical asymptote(s) in the space provided; if none exist write DNE.

$\lim_{x\to 3^+} \frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \infty$ "$\frac{2}{0}$" ← denominator is positive

$\lim_{x\to 3^-} \frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = -\infty$ "$\frac{2}{0}$" ← now denominator is positive

$\lim_{x\to -3} \frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \frac{2}{-6} = -1/3$. no v.a.

Equation(s) of vertical asymptote(s): $x = 3$

(c) Use limits to determine all **horizontal** asymptotes of $g(x)$. Show your work clearly and justify your conclusion using limits. Write the equations of the horizontal asymptote(s) in the space provided; if none exist write DNE.

$\lim_{x\to\infty} \frac{2x^2+6x}{x^2-9} \cdot \frac{1/x^2}{1/x^2}$

$= \lim_{x\to\infty} \frac{2+6/x}{1-9/x^2} = 2$

Equation(s) of horizontal asymptote(s): $y = 2$

$\boxed{6}$ (14 points) Let k be the piecewise defined function below.

$$k(x) = \begin{cases} x^3 + bx^2 & x < 1 \\ \ln(x) - 1 & 1 \le x \le e \\ \dfrac{1}{2-x} & x > e \end{cases}$$

(a) Determine $\lim_{x \to e} k(x)$ or explain why it doesn't exist. (As usual, e is Euler's Constant, $e \approx 2.71828$.)

$$\lim_{x \to e^-} k(x) = \lim_{x \to e^-} \ln(x) - 1 = \ln(e) - 1 = 0$$

$$\lim_{x \to e^+} k(x) = \frac{1}{2-e} \neq 0.$$

Thus $\lim_{x \to e} k(x)$ does not exist (left and right limits are different).

(b) Determine a value for b such that the function $k(x)$ is continuous at $x = 1$, and write your answer in the space below. Show that your choice for b is correct using the *definition of continuity at a point*. (A correct answer will involve writing and computing an appropriate limit or limits.)

$$x = 1: \quad 1^3 + b(1)^2 = \cancel{\ln(1)}^{0} - 1$$

$$1 + b = -1 \qquad b = -2$$

- $\lim_{x \to 1^-} x^3 - 2x^2 = 1 - 2 = -1$

- $k(1) = \ln(1) - 1 = -1$

Thus $\lim_{x \to 1} k(x) = k(1)$. Continuous.

$b = \underline{\ -2\ }$

$\boxed{7}$ (14 points) Consider the function

$$f(x) = \frac{1}{x} + 2x.$$

(a) Using the **definition of the derivative**, find $f'(a)$. Show all your steps using correct notation. No credit will be given if a different method is used. [It is recommended you start by writing the definition of the derivative.]

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

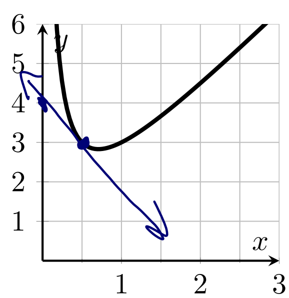
$$= \lim_{x \to a} \frac{\frac{1}{x} + 2x - \left(\frac{1}{a} + 2a\right)}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{a - x}{ax}}{x - a} + \frac{2x - 2a}{x - a}$$

$$= \lim_{x \to a} \frac{-(x-a)}{ax(x-a)} + \frac{2(x-a)}{x-a}$$

$$= \lim_{x \to a} -\frac{1}{ax} + 2 \quad \boxed{= -\frac{1}{a^2} + 2}$$

(b) It is a fact that for this function, $f'(\frac{1}{2}) = -2$. Use this fact to write the equation of the tangent line to the curve at the point with $x = \frac{1}{2}$, and sketch the tangent line on the graph.



$P = (1/2, 3) \quad m = -2$

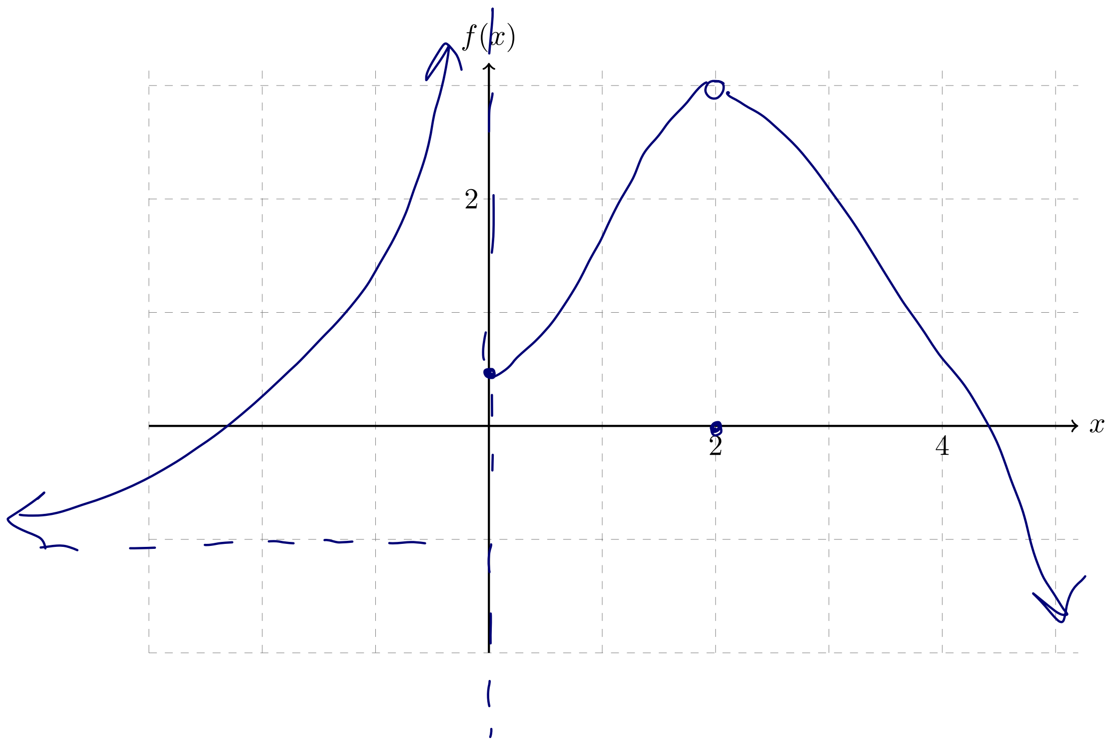
$y - 3 = -2(x - 1/2)$

$y - 3 = -2x + 1$

Equation of Tangent Line: $\boxed{y = -2x + 4}$

$\boxed{8}$ (10 points) Sketch the graph of a function f that satisfies all of the given conditions. Indicate any asymptotes using dashed lines.

$\lim_{x\to-\infty} f(x) = -1$	$\lim_{x\to 0^-} f(x) = \infty$	$\lim_{x\to 2} f(x) = 3$	$\lim_{x\to\infty} f(x) = -\infty$
	$f(0) = 1/2$	$f(2) = 0$	
	$\lim_{x\to 0^+} f(x) = 1/2$		



Extra Credit (5 points) You may choose only ONE of the following two problems. Clearly mark which one you want graded.

EC I Grade This One □

Show that $\lim_{x \to 0} x^2(1 + \sin(1/x)) = 0$. You must clearly explain your work and cite any relevant theorems for full credit.

EC II Grade This One □

Evaluate $\lim_{x \to -\infty} \arctan\left(x^2 - \frac{2x^3}{3\sqrt{1+x^4}}\right)$. Your answer must be preceded by relevant steps and correct notation.