Your Name	Your Signature

Instructor Name	End Time	Desk Number

- The total time allowed for this exam is 90 minutes.
- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

This exam is printed double-sided.

There are problems on both sides of the page!

If you need more space, you may use extra sheets of paper. If you use extra pages:

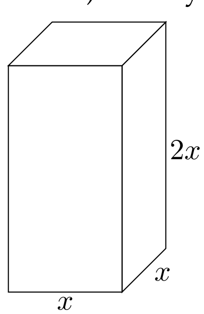
- Put your name on each extra sheet
- Label your work with the problem you're working on
- Write on the exam problem that there is additional work at the end
- Turn in your additional pages at the end of your exam.

$\boxed{1}$ (10 points) Consider the function $g(x) = \dfrac{4}{x} + x$.

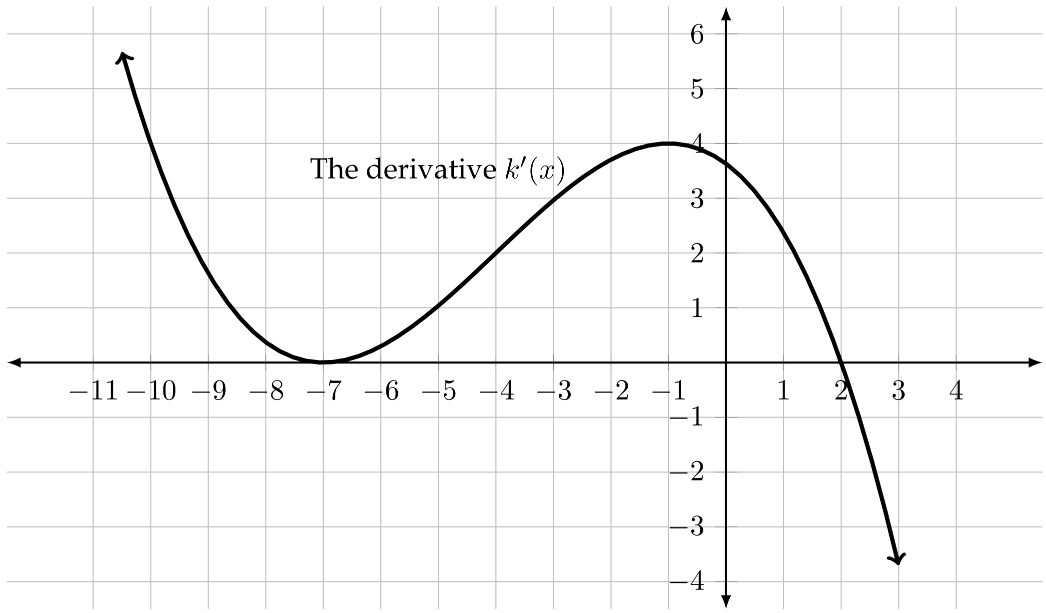
(a) Find the critical number(s) of $g(x)$.

(b) Find the absolute maximum and absolute minimum values of $g(x)$ on the interval $[1/2, 3]$.

$\boxed{2}$ (10 points) A box has a square base and a height that is twice as large as the length of the base. If the length of the base is measured to be 4 cm with an error of ± 1 mm (= 1/10 cm), what is the (absolute) error in the volume of the box? (That is, how much "extra" or "missing" volume is there?) Show your work.



$\boxed{3}$ (14 points) The following graph shows the $\boxed{\text{DERIVATIVE}}$ k' of some function k.



The following questions are about the function $k(x)$, not the graphed $k'(x)$.

(a) Critical points of $k(x)$: ____________________

(b) On what intervals is k increasing or decreasing?

Increasing: ____________________

Decreasing: ____________________

(c) At what values of x does k have a local maximum or minimum? If none, say so.

Local Maxima: $x =$ ____________________ Local Minima: $x =$ ____________________

(d) On what intervals is k concave up or concave down? Use interval notation.

Concave up: ____________________ Concave down: ____________________

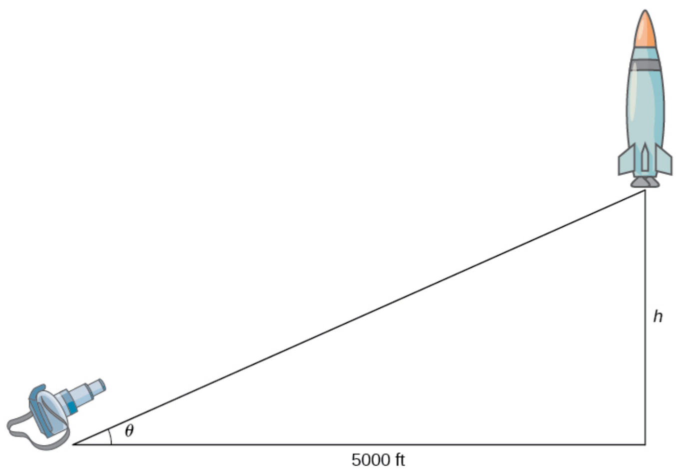
(e) At what values of x does k have inflection points? If none, say so.

Inflection points: $x =$ ____________________

$\boxed{4}$ (14 points) A television camera at ground level is filming the lift-off of a space shuttle that is rising vertically according to the position equation

$$h(t) = 50t^2,$$

where **h** is measured in feet and is **t** measured in seconds (*see picture below*). The camera is 5000 feet from the launch pad.



(a) Find the height and velocity [i.e., change in height] of the shuttle 10 seconds after lift-off.

(b) Find the rate of change in the angle of elevation of the camera (θ) at 10 seconds after lift-off. [Include **units** in your answer]

$\boxed{5}$ (12 points) For each limit:

(i) Write the **form** of the limit AND state whether the **form** is **indeterminate** (include the type).

(ii) Find the limit. If you use a L'Hôpital Rule, indicate it by a symbol (such as **L'H** or **H**) over the equal sign.

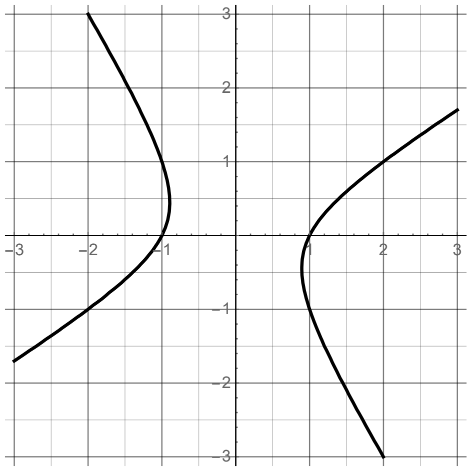
(a) $\lim_{x\to 0} \dfrac{\sin(2x) + 7x^2}{x(x+1)}$ Type:__________

(b) $\lim_{x\to 0} \dfrac{2\cos(\pi x) - 1 + x^2}{2e^{4x}}$ Type:__________

(c) $\lim_{t\to\infty} t \ln\left(1 + \dfrac{3}{t}\right)$ Type:__________

$\boxed{6}$ (10 points) Consider the implicitly defined curve given by

$$x^2 - y^2 = 1 + xy.$$



(a) Show that the point $P = (-1, 1)$ is on the curve. Then **draw and label** the point P in the figure.

(b) Compute y' at P.

(c) Find the equation of the tangent line at P. Then **draw** this tangent line in the figure.

$\boxed{7}$ (14 points) Suppose an open cup in the shape of a cylinder is to be made with surface area 48 in^2. What dimensions (radius and height) will maximize the volume of the cup?

[surface area = $\pi r^2 + 2\pi r h$ and volume = $\pi r^2 h$, where r is the radius of the cup and h is the height.]

8 (16 points) We want to sketch a graph of a function $f(x)$ with certain specified properties.

(a) Fill in the following tables. (You can use words or pictures.)

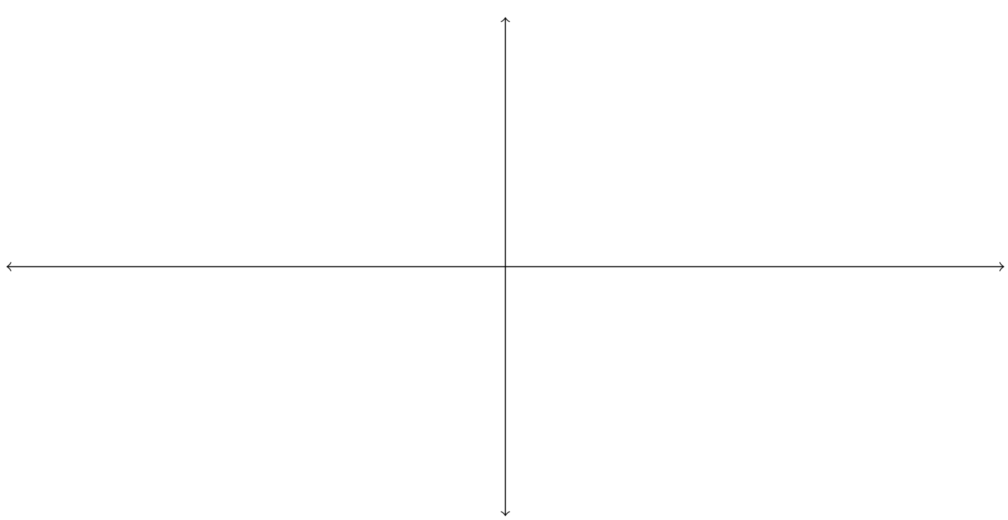
function information	what you conclude about the behavior of f
Domain of f is $(-\infty, \infty)$	
$\lim_{x \to -\infty} f(x) = -2$	
$\lim_{x \to \infty} f(x) = 5$	
$f(0) = 10$	

x	$x < 0$	0	$x > 0$
sign/value of $f'(x)$	$+$	0	$-$
Behavior of $f(x)$			

x	$x < -5$	-5	$-5 < x < 3$	3	$x > 3$
sign of $f''(x)$	$+$	0	$-$	0	$+$
Behavior of $f(x)$					

(b) Sketch the graph of f that has all of the properties listed in the tables (does not need to be drawn to scale). Label/draw **on the graph** the following:

- a point at any local maxima/minima,
- a box at any inflection points,
- a dashed line for any horizontal/vertical asymptotes along with equation,
- tick marks on axes to indicate important x- and y-values.



$\boxed{\text{Extra Credit}}$ (5 points)

Use the Mean Value Theorem to prove that $a - b \leq \sin b - \sin a \leq b - a$ given the interval $[a, b]$.