Math F251 Midterm 1 Fall 2021

Name: ____________________

Section:
- ☐ **F01 (Jill Faudree)**
- ☐ **F02 (James Gossell)**
- ☐ **UX1 (James Gossell)**

Rules:

You have 60 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

The exam is closed book and closed notes.

Calculators are not allowed.

Place a box around your FINAL ANSWER to each question where appropriate.

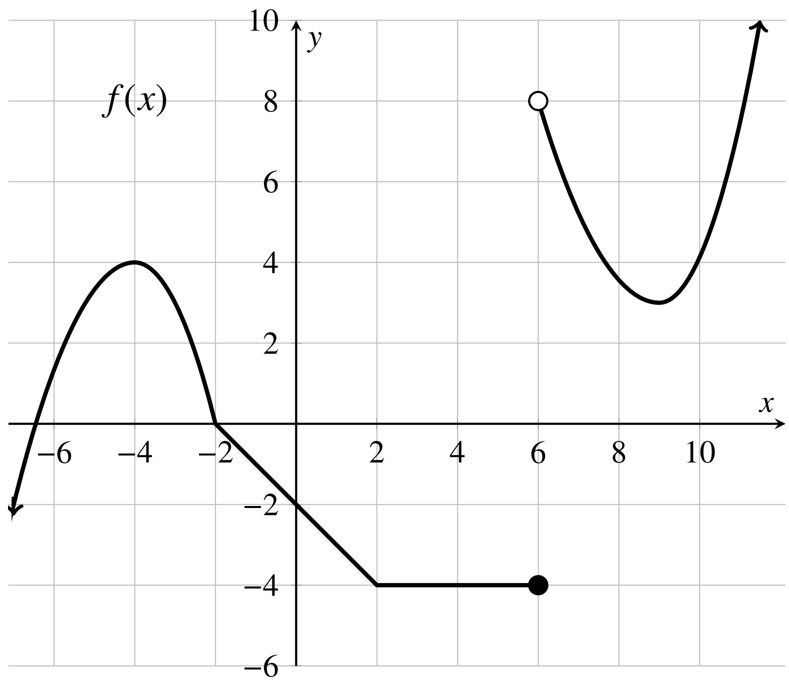
If you need extra space, you can use the back sides of the pages. Please make it obvious when you have done so.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	10	
2	8	
3	12	
4	10	
5	20	
6	12	
7	8	
8	6	
9	14	
Extra Credit	3	
Total	100	

1. 10 points The graph of the function $f(x)$ is given below. The domain of $f(x)$ is $(-\infty, \infty)$. Determine the following. If the value does not exist, write "DNE".



(a) $f(6) =$ -4

(b) $\lim_{x\to 6^+} f(x) =$ 8

(c) $\lim_{x\to 6^-} f(x) =$ -4

(d) $\lim_{x\to 6} f(x) =$ DNE

(e) $\lim_{x\to -4} f(x) =$ 4

(f) $f'(0) =$ -1

(g) List the x-values where $f(x)$ fails to be continuous.

x=6

(h) List the x-values for which $f(x)$ fails to have a derivative.

x= -2,2,6

2. (8 points) Suppose that f and g are differentiable functions with $f(2) = 6$, $f'(2) = 1$, $g(2) = 5$, and $g'(2) = 2$. For $h(x) = f(x) \cdot g(x)$ and $k(x) = \dfrac{f(x)}{g(x)}$, evaluate the following derivatives:

   (a) $h'(2) = f'(2)g(2) + f(2)g'(2) = 1 \cdot 5 + 6 \cdot 2 = 5 + 12 = \boxed{17}$

   (b) $k'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} = \dfrac{(5)(1) - (6)(2)}{(5)^2} = \dfrac{5-12}{25} = \boxed{-7/25}$

3. (12 points) A company's profit in thousands of dollars when x units are produced is given by

$$P(x) = -0.001x^2 + 0.22x - 42.$$

   (a) Observe that $P(1200) = 78$. Interpret this fact in the context of the problem. To earn full credit your answer should be a complete sentence and must include units.

   When the company produces 1200 units, its profits are 78,000 dollars.

   (b) Observe that $P'(1200) = -0.02$. Interpret this fact in the context of the problem. To earn full credit your answer should be a complete sentence and must include units.

   When the company produces 1200 units, its rate of change of profit is -0.02 thousands of dollars per unit. (or a -20 dollars per unit)

   (c) Suppose the company is currently producing 1200 units. Should the company increase production if it wants to increase profits? **Justify** your answer.

   No. Since the rate of change of profit with respect to units produced is negative, producing the 1201st unit would actually decrease profit.

4. (10 points) Find the derivative of $f(x) = \dfrac{4}{x+1}$ **using the definition of the derivative**. No credit will be awarded for using another method.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{4}{x+h+1} - \frac{4}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{4(x+1) - 4(x+h+1)}{(x+h+1)(x+1)}\right) = \lim_{h \to 0} \frac{1}{h}\left(\frac{\cancel{4x}+\cancel{4}-\cancel{4x}-4h-\cancel{4}}{(x+h+1)(x+1)}\right)$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left(\frac{-4\cancel{h}}{(x+h+1)(x+1)}\right) = \lim_{h \to 0} \frac{-4}{(x+h+1)(x+1)}$$

$$= \frac{-4}{(x+0+1)(x+1)} = \boxed{\frac{-4}{(x+1)^2}}$$

5. (20 points) Evaluate the following limits. Give the most complete answer; if the limit is infinite, indicate that with ∞ or $-\infty$. If a value does not exist, write DNE.

(a) $\lim\limits_{x\to 5} \dfrac{5x - x^2}{20 + x - x^2} = \dfrac{25-25}{20+5-25} = \dfrac{0}{0}$ ← factor

$$\lim_{x\to 5} \frac{x(5-x)}{(5-x)(4+x)} = \lim_{x\to 5} \frac{x}{4+x} = \boxed{\frac{5}{9}}$$

(b) $\lim\limits_{x\to 7^+} \dfrac{\overset{+}{x + 12}}{\underset{-}{7 - x}} = \dfrac{19}{0}$ ← This limit is infinite.

Ans: $\lim\limits_{x\to 7^+} \dfrac{x+12}{7-x} = \boxed{-\infty}$

As $x \to 7^+$, $7-x \to 0^- < 0$

and $x+12 \to 19 > 0$

(c) $\lim\limits_{h\to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \dfrac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})} = \lim\limits_{h\to 0} \dfrac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x+0}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

(d) $\lim\limits_{t\to 0^-} \left(\dfrac{t}{2} + \dfrac{6}{t-2}\right) = \dfrac{0}{2} + \dfrac{6}{-2} = \boxed{-3}$

6. (12 points) Find $f'(x)$ for each of the following expressions. You do not need to simplify.

   (a) $f(x) = 6x^3 + 5x^2 - \sqrt{x} + \sqrt{\pi}$ $x^{1/2}$

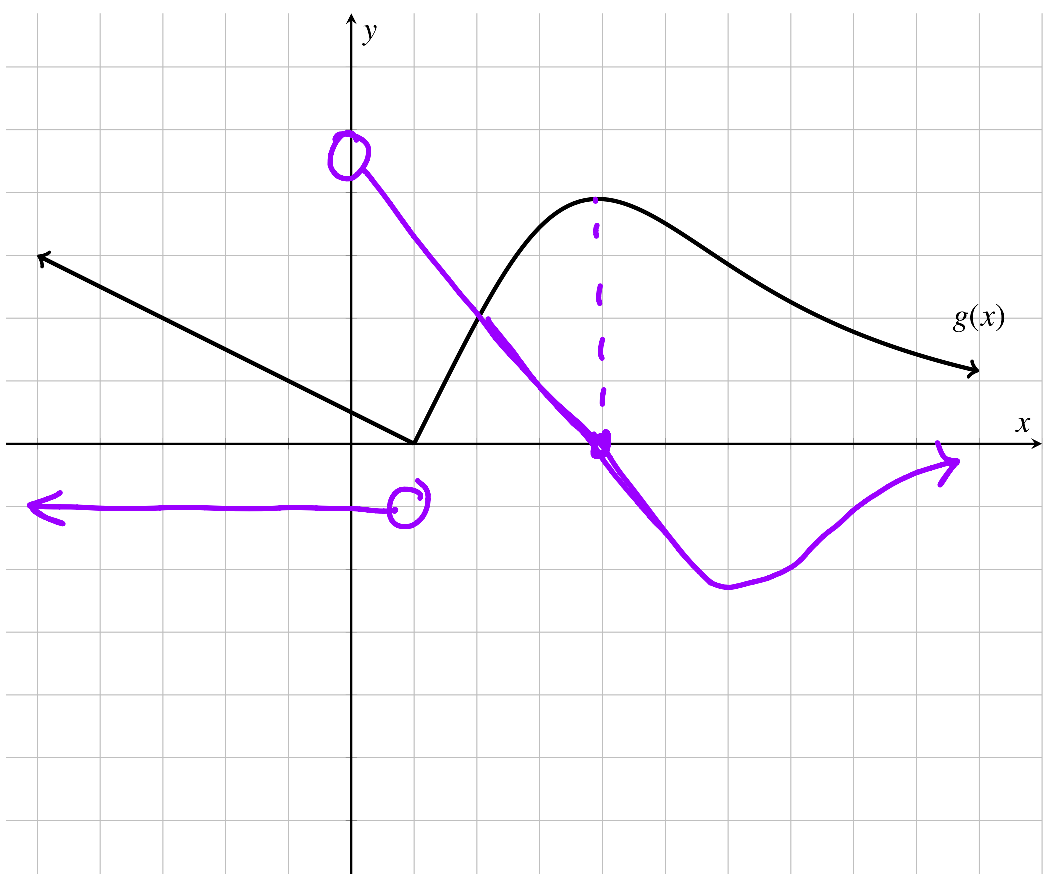
$$\boxed{f'(x) = 18x^2 + 10x - \frac{1}{2}x^{-1/2} + 0 = 18x^2 + 10x - \frac{1}{2\sqrt{x}}}$$

   (b) $f(x) = \frac{2x-4}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(2) - (3x-4)(2x)}{(x^2+1)^2} = \frac{2x^2+2-6x^2-8x}{(x^2+1)^2}$$

$$= \boxed{\frac{-4x^2-8x+2}{(x^2+1)^2}}$$

7. (8 points) The graph of $g(x)$ is graphed below. Sketch the graph of its derivative $g'(x)$.

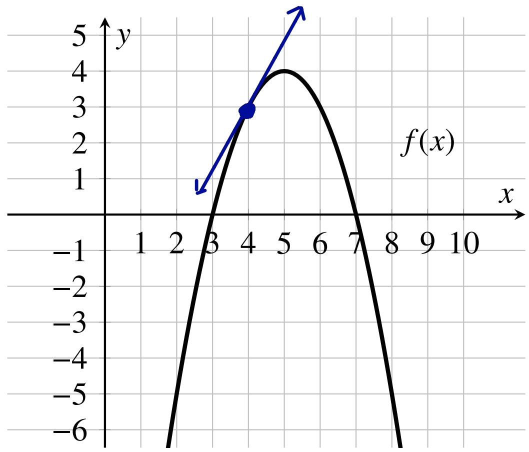


8. (6 points) The questions below reference the function $f(x) = (3 - x)(x - 7)$. $= -x^2 + 10x - 21$

   (a) Find the slope of the line tangent to at $x = 4$.

   $f'(x) = -2x + 10 \quad f'(4) = -2 \cdot 4 + 10 = 2$

   (b) Sketch the tangent line to $f(x)$ at $x = 4$ on the graph of $f(x)$ below.



9. (14 points) Patrick throws a football straight up from the surface of the moon. While the football is in the air, its height in meters after t seconds is $h(t) = t(16 - 0.8t)$. Answer the following questions **including units in your answers**. $= 16t - 0.8t^2$

   (a) What is the initial velocity of the football?

   $h'(t) = 16 - 1.6t$
   $h'(0) = \boxed{16 \text{ m/s}}$

   The initial velocity of the football is 16 m/s.

   (b) When will the football reach its highest point?

   Set $v = h' = 0$: $16 - 1.6t = 0$ So $\boxed{t = 10s.}$

   The football reaches its highest point 10 seconds after it is thrown.

   (c) When will the football hit the ground?

   Set $h = 0$.
   $0 = t(16 - 0.8t)$

   So $t = 0$ (start) or $16 - 0.8t = 0$
   So $\boxed{t = 20 \text{ sec.}}$

   (and (!!) by symmetry it is 2·times my answer from part b :))

   (d) What is the acceleration due to gravity on the moon?

   $a = v' = s'' = \boxed{-1.6 \text{ m/s}^2}$

   The acceleration due to gravity on the moon is -1.6 m/s/s

10. **Extra Credit** (3 points)Use the Intermediate Value Theorem to **prove** that there exists some x-value in the interval $(0, 100)$ such that the function $g(x) = e^{-x} + \sqrt{x} + 8$ takes the value $y = 10$. Full credit will only be given if the solutions provides and explanation using complete sentences.

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ANSWER: The function $g(x)$ is continuous and $g(0) = 9$ and $g(100) = e^{-100} + 10 + 8 = 18 + \frac{1}{e^{100}} > 18$.

Since $y=10$ is between $y=9$ and $y = 18 + \frac{1}{e^{100}}$, the IVThm says there is some x-value in $(0,100)$ such that $g(x) = 10$.