Fall 2023 # Midterm 1 **Math F251X**

Name: ____________________

Rules:

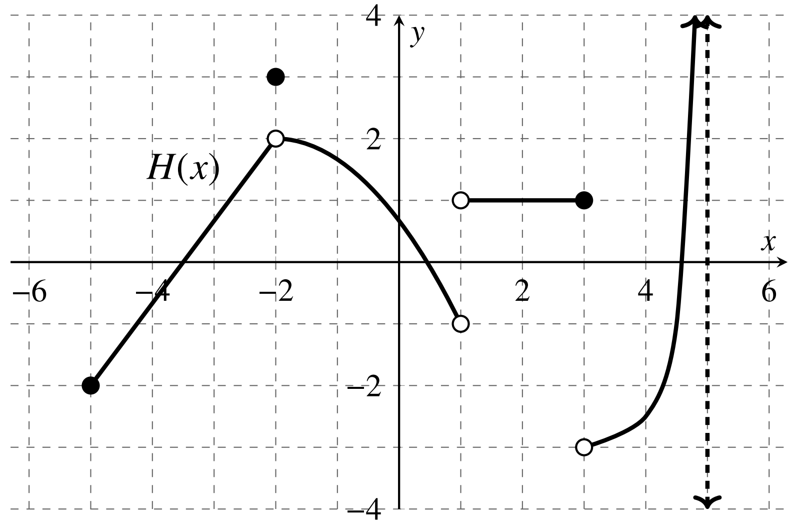
- Partial credit will be awarded, but you must show your work.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Calculators are not allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	16	
2	12	
3	10	
4	12	
5	12	
6	8	
7	8	
8	10	
9	12	
Extra Credit	5	
Total	100	

1. (16 points)

The **entirety** of a function $H(x)$ is shown below. Use the graph of $H(x)$ to answer each question below. If a limit is infinite, indicate that with ∞ or $-\infty$. If a value does not exist or is undefined, write **DNE**.



a. What is the domain of $H(x)$? Write your answer in interval notation.

domain = ____

b. $\lim_{x \to -2} H(x) =$ ______

c. $H(-2) =$ ______

d. $\lim_{x \to 1} H(x) =$ ______

e. $\lim_{x \to 5^-} H(x) =$ ______

f. $\lim_{x \to 3^+} H(x) =$ ______

g. $H(3) =$ ______

h. $H'(-4) =$ ______

i. $H'(2) =$ ______

j. $\lim_{x \to -2^+} H'(x) =$ ______

k. List the values of x $\boxed{\text{in the domain of } H}$ where $H(x)$ is NOT continuous.

$x =$ ____

2. (12 points)

Compute the following limits. If the limit does not exist, write **DNE** and a few words about why it does not exist. If the limit increases without bound, write ∞ or $-\infty$.

a. $\lim_{x \to 3} \dfrac{2x^2 - 5x - 3}{x^2 + x - 12}$

b. $\lim_{t \to 1^-} \dfrac{(t-2)(3t+5)}{(t+1)(t-4)}$

c. $\lim_{x \to 5} \dfrac{\sqrt{x} - \sqrt{5}}{x - 5}$

d. $\lim_{x \to 2^-} \dfrac{2x^2 - x - 3}{4x - 8}$

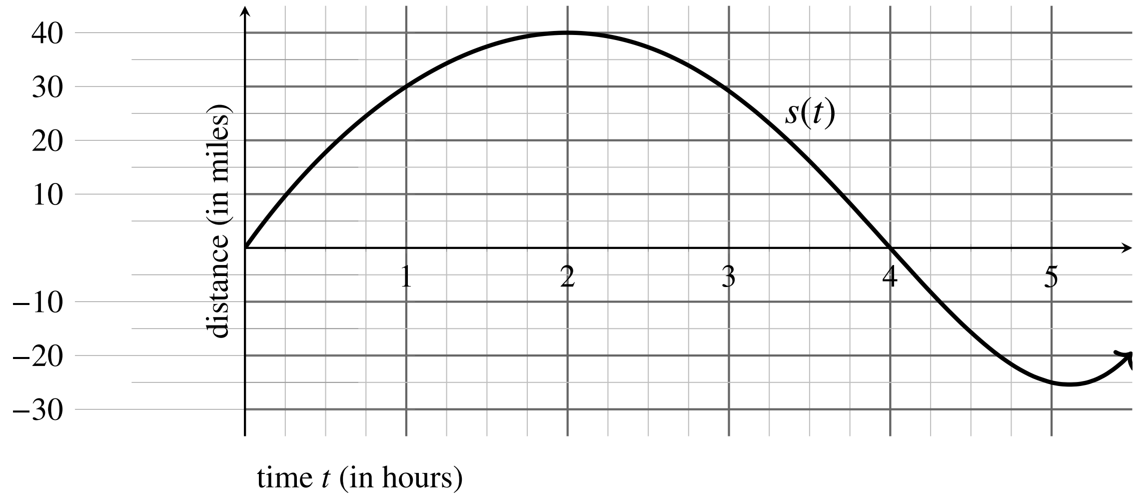
3. (10 points)

Consider the function
$$f(x) = \frac{1}{6 - x}.$$

Find $f'(5)$ using the **limit definition of the derivative** and show your work using all appropriate notation. No credit will be awarded for using other methods. Begin by writing down the limit definition of the derivative.

4. (12 points)

A police station is located on a straight east-west road. At 9:00 AM, a patrol car leaves the station going east. The function $s(t)$ gives the position of the car, in miles, t hours after 9:00 AM. A positive value for $s(t)$ means that the car is to the east of the station. The graph of $s(t)$ is shown below.



a. Determine $s(1)$ and interpret this value in the context of the problem. Your answer should be a sentence and it should include units.

b. Find the average rate of change between $t = 1$ and $t = 5$, and interpret this value in the context of the problem. Your answer should be a sentence and it should include units.

c. Estimate $s'(1)$. Show some work.

d. Explain in simple terms what $s'(1)$ indicates in the context of the problem. Your answer should be a sentence and it should include units.

e. In the context of the problem, what happens to the patrol car at $t = 4$?

f. What is $s'(2)$, and what does that mean in the context of the problem?

5. (12 points)

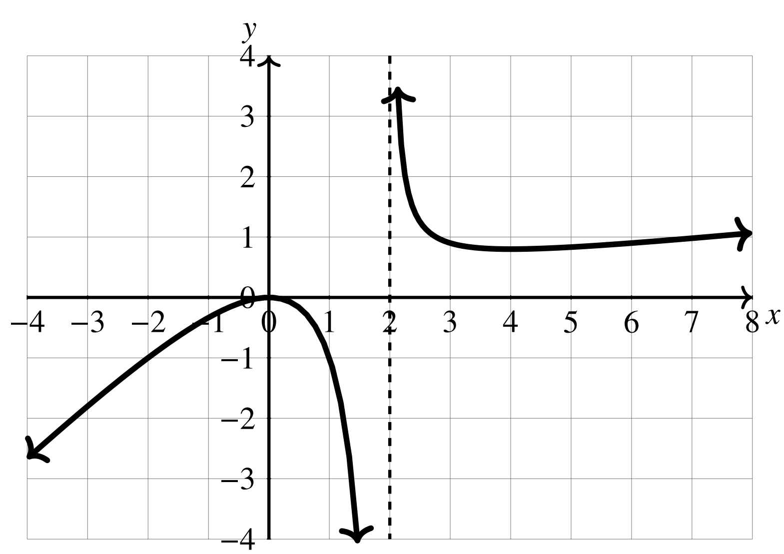
Consider the function $f(x) = \dfrac{x^2}{x-2}$.

a. Find $f'(x)$. Use whatever method you like. Show your work.

b. Use your answer from part (a) to determine the slope of the line tangent to $f(x)$ at the point $P(1, -1)$.

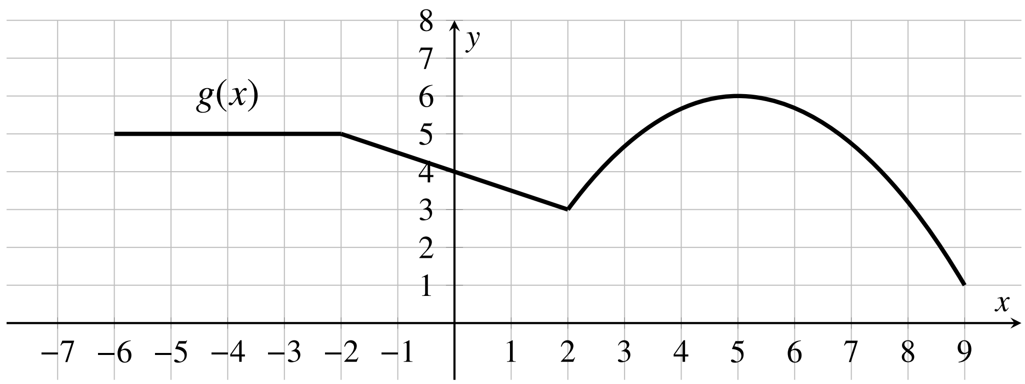
c. Write down an equation for the tangent line at the graph of $f(x)$ at the point $P(1, -1)$.

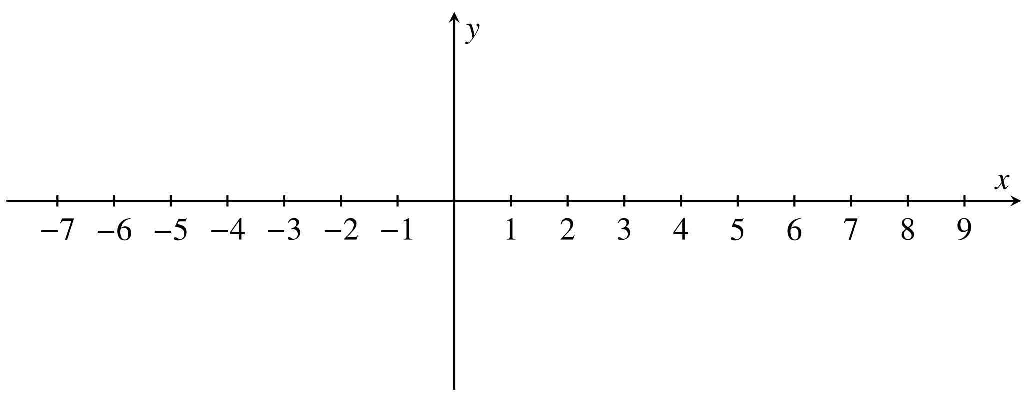
d. Clearly draw the tangent line at the graph of $f(x)$ at the point $P(1, -1)$ on the graph below and label it as "tangent".



6. (8 points)

A function $g(x)$ is shown. Sketch the derivative $g'(x)$ on the second set of axes. Indicate any asymptotes the derivative might have using dashed lines, and indicate any points where the derivative is undefined using open circles.





7. (8 points)

Consider the function $f(x) = 2\sin x - x$. Determine all x-values on the interval $[0, 2\pi]$ for which $f(x)$ has a **horizontal tangent line**.

8. (10 points)

Let

$$f(x) = \begin{cases} \dfrac{3x^2 + x}{x} & x < 0 \\ 2 & x = 0 \\ \sqrt{x} + e^x & x > 0 \end{cases}$$

a. Evaluate $\lim_{x \to 0^-} f(x)$. Show supporting work.

b. Evaluate $\lim_{x \to 0^+} f(x)$. Show supporting work.

c. Evaluate $f(0)$.

d. Based on your answers to parts (a), (b) and (c), **check the true statement(s) below**:

- ☐ f is continuous at $x = 0$.
- ☐ f has a removable discontinuity at $x = 0$.
- ☐ f has a jump discontinuity at $x = 0$.
- ☐ f has an infinite discontinuity at $x = 0$.
- ☐ None of the above.

9. (12 points)

For each of the following functions, compute the derivative.

You do not need to simplify your answers.

a. $y = 11x^3 - \frac{4x}{3} + x^2 + x^{5/3}$

b. $a(\theta) = \theta^3 \cos(\theta)$

c. $f(t) = t\sqrt{t} - \dfrac{1}{8t^4} + \sqrt{2}$

$\boxed{\text{Extra Credit}}$ (5 points) Use the Intermediate Value Theorem to show that the equation $x^3 = 2^x$ has a solution for some x-value. Justify your answer with words (as well as computations).