Fall 2024 **Math F251X**

Calculus I: Final Exam

Name: ____________________

Section:
- ☐ **9:15 (James Gossell)**
- ☐ **11:45 (Jill Faudree)**
- ☐ **11:45 (Leah Berman)**
- ☐ **Online (James Gossell)**

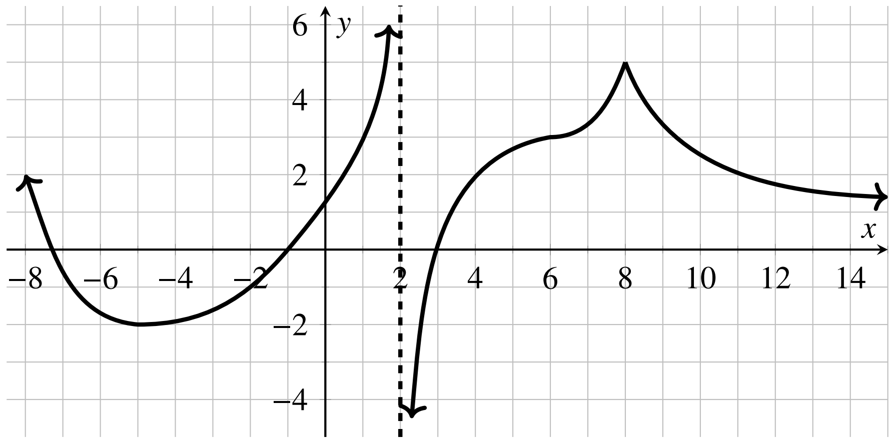
Rules:

- Partial credit will be awarded, but you must **show your work**.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Calculators are **not allowed**.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.
- You have two hours to complete the exam.

Problem	Possible	Score
1	16	
2	6	
3	10	
4	6	
5	8	
6	9	
7	11	
8	9	
9	16	
10	9	
Extra Credit	(5)	
Total	100	

1. (16 points)

Consider the graph of the function $f(x)$ shown below, and answer the following questions. You must give the most complete answer; if the value is infinite, write $+\infty$ or $-\infty$.



a. $\lim_{x\to 2^+} f(x) =$ ______

b. $\lim_{x\to 8} f(x) =$ ______

c. $\lim_{x\to 2^+} f'(x) =$ ______

d. List all x-values where the derivative $f'(x)$ is not defined. ______

e. Estimate $f'(10) =$ ______. **Explain** how you computed your estimate.

f. List all intervals where $f'(x) > 0$. ______

g. As what x-values does $f(x)$ have...(If none write "none").

A local maximum? ______ A local minimum? ______.

h. The line $y = 1$ is a horizontal asymptote of $f(x)$. Fill in a statement about a limit that corresponds to this fact.

$$\lim_{\square} \square = \square$$

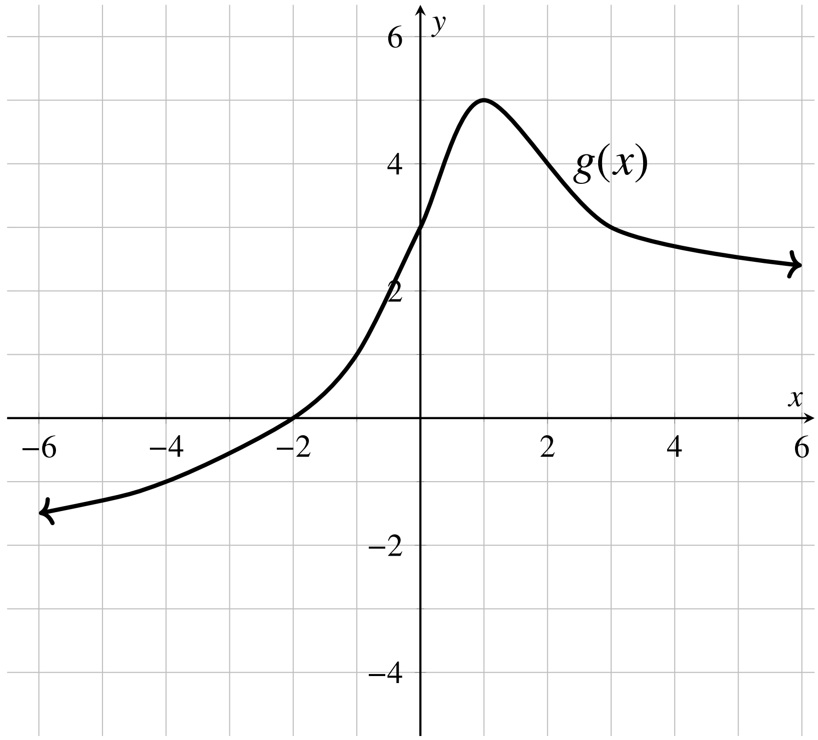
i. Does f have an(y) inflection point(s)? If so, list it/them, if not write "none."

Inflection point(s): $x =$ ______

j. On what interval(s) is $f(x)$ concave down? ______

2. (6 points)

The graph of a function $g(x)$ is shown below.



Define a new function $A(x) = \int_{-4}^{x} g(t)\,dt$.

a. On the interval $(-4, 4)$, does $A(x)$ have a local maximum or a local minimum? If so, give the corresponding x-value and **explain** your answer. If not, explain why not.

b. **Estimate** $A(0) =$_____ and clearly explain what calculation you did to arrive at that estimation. (You may want to draw on the graph as part of your explanation.)

c. Determine $A'(2) =$ _____.

3. (10 points)

Compute the following **integrals**. Give the most general answer, and show your work. Clearly indicate any substitutions you use in such a way that someone else can follow your work.

a. $\displaystyle\int x\cos(x^2) + \frac{1}{\sqrt{1-x^2}}\,dx$

b. $\displaystyle\int \left(3t^{\frac{2}{7}} - 8t^{-1} + e^{5t-4} + \sin\left(\frac{\pi}{6}\right)\right)\,dt$

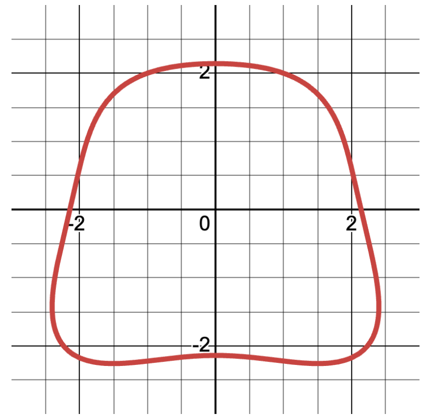
4. (6 points)

Use linearization to estimate $\sqrt{98}$. Show your work, and write your answer as a single decimal or fraction.

5. (8 points)

Below is the graph of the curve $x^4 + y^4 + 2x^2y = 21$.

a. Find $\dfrac{dy}{dx}$ for the curve $x^4 + y^4 + 2x^2y = 21$.



b. **Write an equation** of the line tangent to the curve $x^4 + y^4 + 2x^2y = 21$ at the point $(-1, 2)$.

Tangent line equation: ____________________

c. **Draw** the tangent line on the figure above.

6. (9 points)

The Mars rover drops a rock off of a 61 foot cliff. While in free fall, the rock's **velocity** after t seconds is given by the function $v(t) = -12.2t$ feet per second.

a. Evaluate the integral $\int_0^2 v(t)\,dt$ and write a sentence to interpret its meaning in the context of the problem. Include units in your answer.

b. Write a function $h(t)$ that gives the rock's height above the ground after t seconds.

c. What is the **acceleration** due to gravity on Mars? Include units in your answer.

7. (11 points)

A box has a square base and an open top. The material for the base costs \$4 per square meter and the material for the sides costs \$1 per square meter. Suppose the width of the base of the box is x meters and its height is y meters.

a. What is the total cost, C, of the box?

b. What is the volume, V, of the box?

c. Suppose you have \$24 to spend on the materials for the box.

(i) Write the volume V as a function of one variable and pick a **domain** for this function.

(ii) Determine the **dimensions** of the box of **largest** possible volume that fits within your budget.

dimensions (include units): $x =$ __________ $y =$ __________

(iii) **Justify** that your dimensions give the largest volume, using calculus. As part of your justification, **write the name** of the test you are applying (first derivative test, second derivative test, closed interval/extreme value theorem, some other test).

8. (9 points)

Compute the following **limits**. Show your work clearly. Make sure you use **limit notation** where required; an answer that does not use proper notation will not receive full credit. Use = to show things are equal. If you use L'Hôpital's rule, write $\overset{H}{=}$ or $\overset{L'H}{=}$ to indicate where you are applying it.

a. $\displaystyle\lim_{\theta\to\pi} \frac{1+\cos(\theta)}{\sin^2(\theta)}$

b. $\displaystyle\lim_{h\to 0} \frac{\ln(e+h)-1}{h}$

c. $\displaystyle\lim_{x\to\infty} \frac{-10x^3+7x^2}{2x^3+3x-5}$

9. (16 points)

At 3:00 pm ($t = 0$), workers discover a break in an oil pipeline. The rate at which oil is flowing from the break is given by $r(t) = \dfrac{10t}{4 + t^2}$, where r is measured in gallons per hour and t is measured in hours.

a. Find $r(1)$. Write a sentence that someone who has not taken calculus could understand to explain the meaning of $r(1)$. Include units.

b. Determine how much oil flowed out of the pipeline between 3:00pm and 5:00pm. Include units in your answer.

c. At 4:00pm, is the rate of flow increasing, decreasing or staying the same? Explain how you know.

d. At what time is the rate of flow of the oil at its maximum? (Give your answer as a time.)

10. (9 points)

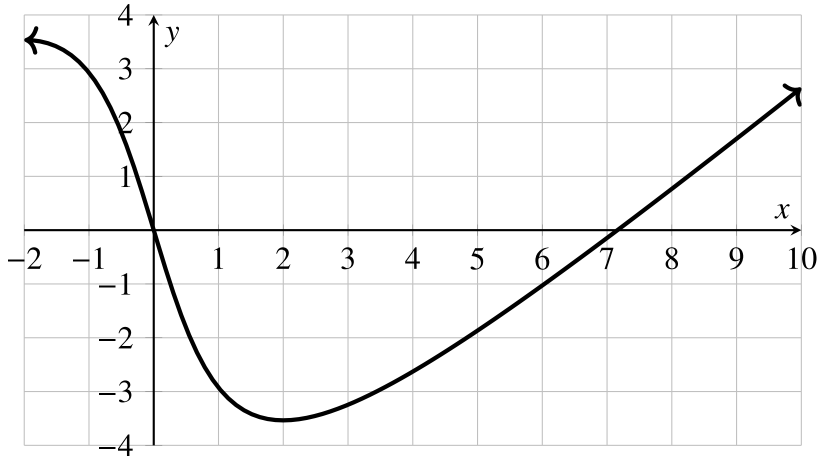
Compute the following **derivatives**. Show your work. You do NOT need to simplify your answer. Your answer should start $f'(x)$, $\frac{df}{dx}$ etc.

a. $h(x) = 4\sec\left(5x^{-2}\right)$

b. $f(x) = x^{-3} + \sqrt[3]{(2x)} - 3^x + 3$

c. $g(x) = \ln\left(\frac{x^6 e^x}{\sqrt{x}}\right)$

$\boxed{\textbf{Extra Credit}}$ (5 points) A portion of the graph of the function $f(x) = x - 5\arctan(x)$ is shown below.



a. Suppose Newton's method is used to find an approximate solution to $f(x) = 0$ from an initial guess of $x_1 = 4$. **Sketch** on the graph how the next approximation x_2 will be found, **labeling** its location on the x-axis.

b. If your starting guess is $x_1 = 4$, **compute** x_2. Show your work. **You do not need to simplify completely**, but your answer should be in a form where typing it into a calculator would compute a numerical value.

c. What happens if you try to apply Newton's method with a starting guess of $x_1 = 2$?