Fall 2024 **Math F251X**

Calculus 1: Midterm 2

Name: ____________________

Section:
- ☐ **9:15am (James Gossell)**
- ☐ **11:45am (Jill Faudree)**
- ☐ **11:45am (Leah Berman)**
- ☐ **async (James Gossell)**

Rules:

- Partial credit may be awarded, but you must show your work.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Calculators are **not** allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	12	
2	10	
3	12	
4	11	
5	12	
6	12	
7	12	
8	9	
9	10	
Extra Credit	5	
Total	100	

1. (12 points)

Evaluate the following limits. **Show your work**, uncluding appropriate use of limit notation. If you use L'Hôpital's rule, you must indicate where you are using it by writing $\stackrel{H}{=}$ or $\stackrel{L'H}{=}$ or something similar. Use ∞ or $-\infty$ where appropriate, and if the limit does not exist, write DNE and provide a justification.

a. $\displaystyle\lim_{x\to\infty} \frac{2x - 4x^3}{x^3 - 4x^2 - 6}$

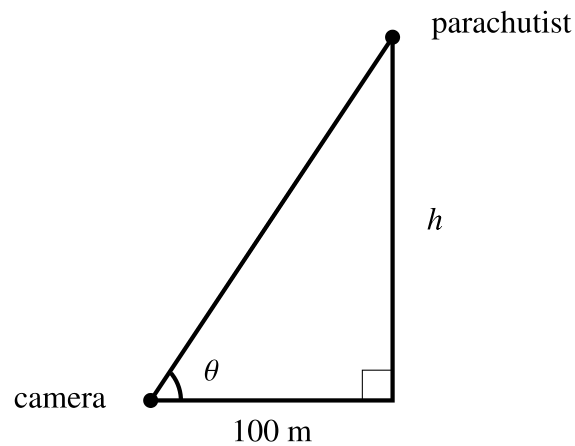
b. $\displaystyle\lim_{t\to 3} \frac{e^{t-3} - t + 2}{t^2 - 6t + 9}$

c. $\displaystyle\lim_{\theta\to 0} \frac{2\sin(\theta) - 2}{1 - \theta - e^{\cos(\theta)}}$

2. (10 points)

A camera at ground level is 100 meters from the landing site of a parachutist who is landing vertically. Let h be the height of the parachutist above the ground and let θ be the angle of elevation formed between the camera lens and the ground. (See figure.)

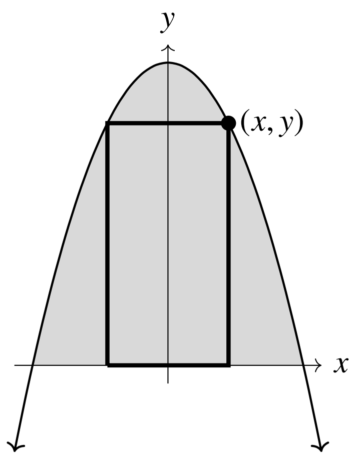
a. Find an equation relating h and θ.



b. Suppose the height of the parachutist decreases at a constant rate of 5 meters per second. At what rate does the angle θ decrease when the parachutist is 200 meters in the air? **Answer the question with a complete sentence, including units.**

3. (12 points)

We want to determine the dimensions of the rectangle of maximum area that is inscribed between the parabola $y = 5 - x^2$ and the x-axis. Assume the base of the rectangle is on the x-axis. (See figure below; the rectangle should be inside the shaded area.)



a. Find an expression for the area A of the rectangle as a function of one variable.

b. State the appropriate domain of the area function given the context of the problem.

c. Use Calculus to determine where the area is maximized. Justify your conclusion with work.

d. Answer the question:
The dimensions of the rectangle with largest area are ____________________

4. (11 points)

Consider the function $g(x) = \dfrac{x^{2/3}}{x-5}$. After simplification, $g'(x) = \dfrac{-(x+10)}{3x^{1/3}(x-5)^2}$.

a. What are the critical numbers of $g(x)$?

b. At what x-values does $g(x)$ have local maximum(s)? At what x-values does g(x) have local minimum(s)? Clearly show work to **justify** your answers.

Local maximum(s): $x =$ ________________ Local minimum(s): $x =$ ________________

(If none, write "none".)

c. Does $g(x)$ have any horizonal asympotes? If it does, write the equations of any horizontal asymptote(s) of $g(x)$, and justify each answer by writing a limit. If it doesn't, explain why $g(x)$ does not have any horizontal asymptotes and write "none".

Horizontal asymptote equation(s): ________________________________

(If none, write "none".)

5. (12 points)

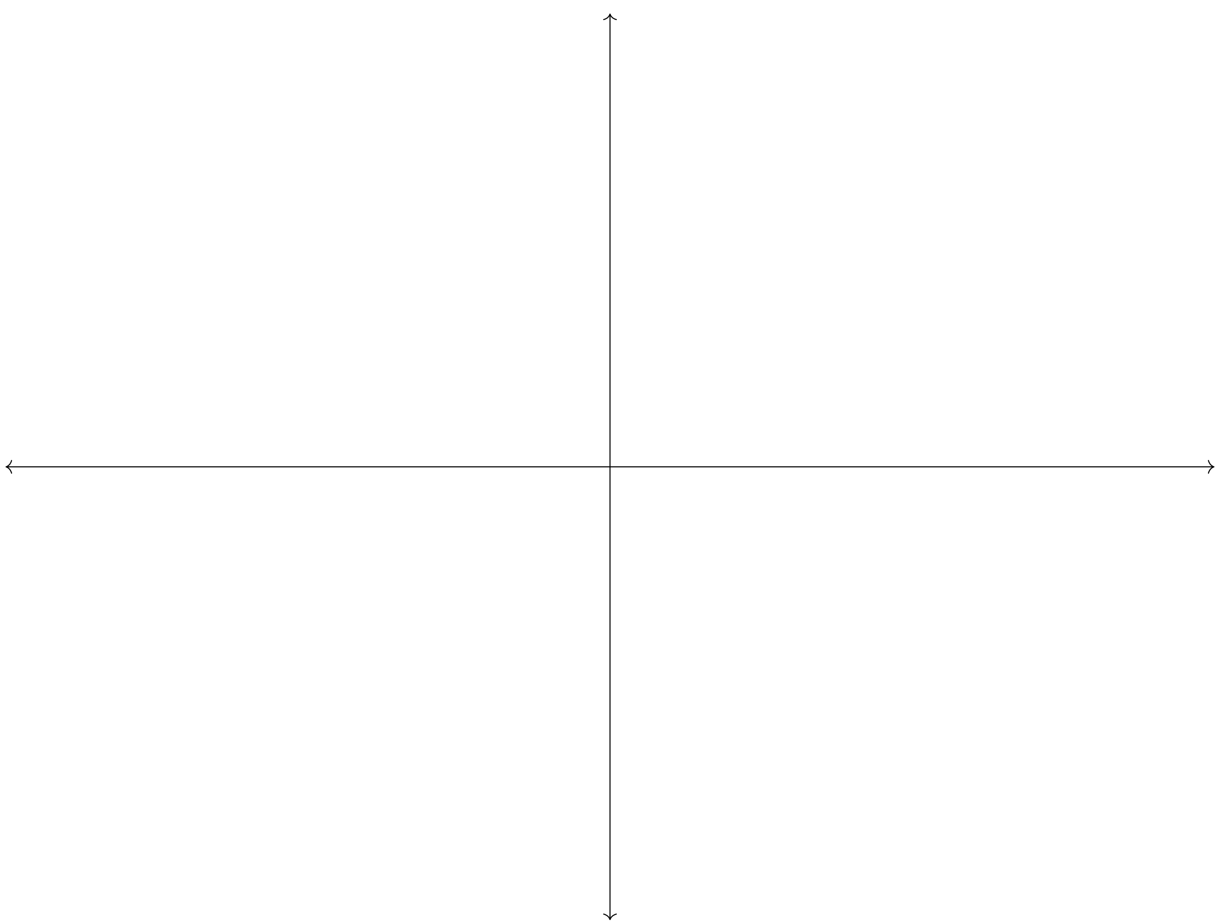
Sketch a graph of a function $f(x)$ that satisfies all of the following properties.

After drawing the graph:

- **Label** on the graph the following things, if they exist, by drawing a point on the graph and labeling: any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP
- Draw any horizontal and vertical asymptotes with dashed lines and **label** them with their equation.
- Mark any important x-values and y-values on the x- and y-axes.

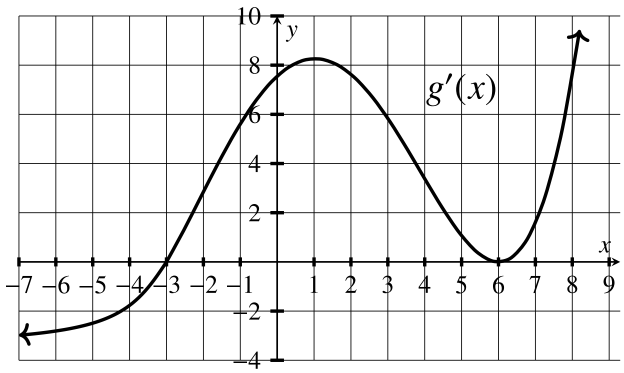
Properties:

- Domain is $(-\infty, 1) \cup (1, \infty)$
- $f(4) = -1$ and $f'(4) = 0$
- $\lim_{x \to 1^+} f(x) = \infty$
- $f'(x) < 0$ on $(-\infty, 1) \cup (1, 4)$
- $f'(x) > 0$ on $(4, \infty)$
- $f''(x) < 0$ on $(-\infty, 1) \cup (6, \infty)$
- $f''(x) > 0$ on $(1, 6)$



6. (12 points)

The graph shown below is the graph of the $\boxed{\textbf{derivative } g'(x)}$ of a function $g(x)$. Answer the following questions about the **original** function $g(x)$.



a. Determine the critical numbers of $g(x)$. (Notice that $g(x)$ is $\boxed{\textbf{not}}$ shown on the graph!)

b. Determine the intervals where g is **increasing** and where g is **decreasing**. If none write "none".

Increasing: ______________________ Decreasing: ______________________

c. Fill in the blanks (if none, write "none"):

$g(x)$ has (a) local maximum(s) at $x =$ __________ and (a) local minimum(s) at $x =$ __________.

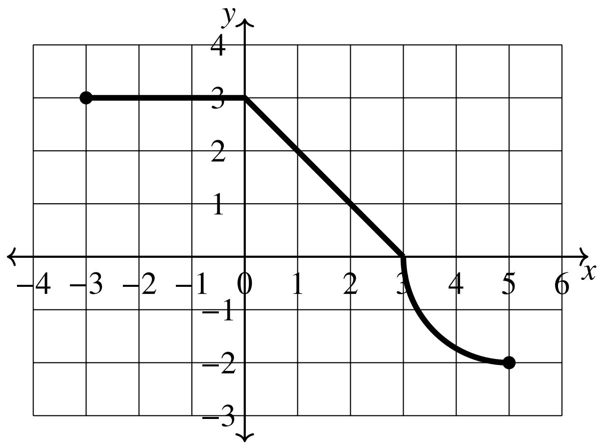
d. Find all intervals where g is **concave up** and where g is **concave down**. (If none write "none".)

Concave up: ______________________ Concave down: ______________________

e. Fill in the blanks: $g(x)$ has (an) inflection point(s) at $x =$ __________. (If none, write "none".)

7. (12 points)

The graph of the function $G(x)$ is below. The function $G(x)$ has domain $[-3,5]$ and the portion of the graph on the interval $[3,5]$ is one quarter of a circle of radius 2 centered at the point $(5,0)$.



a. Determine $\displaystyle\int_{-2}^{3} G(x)\,dx$.

b. Determine $\displaystyle\int_{-3}^{5} G(x)\,dx$.

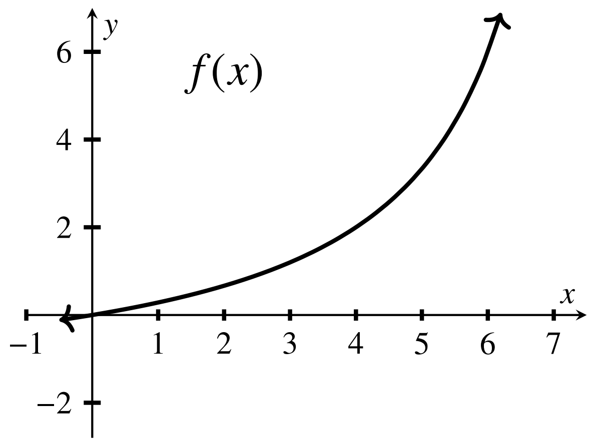
c. Determine $\displaystyle\int_{-3}^{5} 2G(x) + 3\,dx$ (Hint: Use part (b) above.)

8. (9 points)

Consider the function

$$f(x) = \frac{2x}{8-x}.$$

A portion of the graph of this function is shown to the right.



a. Compute R_3 on the interval $[0, 6]$. That is, approximate $\int_0^6 f(x)\,dx$ using 3 right-hand rectangles. **Draw the rectangles on the graph.**

b. List **two distinct strategies** to compute a more accurate approximation of $\int_0^6 f(x)\,dx$.

9. (10 points)

Evaluate the indefinite integrals below. (Give the most generic answer.)

a. $\int \left(3x^3 + \cos(x) - e^x + \sqrt{5}\right)\,dx$

b. $\int \frac{1 + x^{\frac{1}{3}} + x^4}{x}\,dx$

$\boxed{\textbf{Extra Credit}}$ (5 points)

A population of bacteria can be modeled by the function $P(t) = t^{k/t}$, where t is time, measured in hours, P is the number of bacteria, measured in thousands, and k is a fixed positive constant.

a. Compute $\lim_{t\to\infty} P(t)$.

b. Interpret this limit by writing a complete sentence, including units, using the context of the model.