Math F251 Midterm 1 Spring 2020

Name: Solutions

Section:
- ☐ F01 (Faudree)
- ☐ F02 (Bueler)
- ☐ UX1 (Van Spronsen)

Rules:

You have 60 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

No calculators, books, notes, or other aids are permitted.

Circle your final answer to each question where appropriate.

If you need extra space, you can use the back sides of the pages. (Clearly label any work you want graded.)

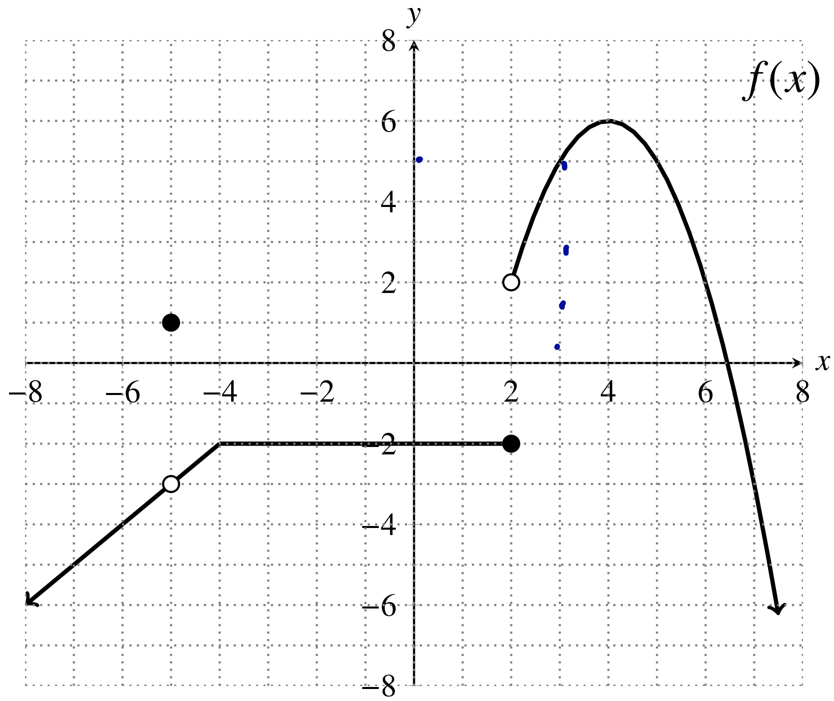
Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	12	
2	20	
3	10	
4	6	
5	15	
6	10	
7	15	
8	12	
Extra Credit	3	
Total	100	

1. (12 points)

Use the graph of $f(x)$ to answer the following questions.



a. Fill in the blanks below:

$\lim_{x\to -5} f(x) =$ -3 $\qquad \lim_{x\to 2^+} f(x) =$ 2 $\qquad \lim_{x\to 2} f(x) =$ DNE

$f(-5) =$ 1 $\qquad f(2) =$ -2 $\qquad \lim_{x\to 3} f(x) =$ 5

b. State the x-values for which f is **not** continuous.

$x = -5, 2$

c. State the x-values for which f is **not** differentiable.

$x = -5, -4, 2$

2. (20 points)

Compute the following limits, or explain why the limit does not exist. (Show appropriate work and use proper limit notation for full credit.)

a. $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \lim_{x\to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)} = \lim_{x\to 0} \frac{x}{x(\sqrt{x+4}+2)}$

form $\frac{0}{0}$

$= \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{4}$

b. $\lim_{x\to\infty} \frac{x-4x^2}{2x^2-5} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{\frac{1}{x}-4}{2-\frac{5}{x^2}} = \frac{-4}{2} = -2$

c. $\lim_{y\to\pi/2^+} \frac{\sin(y)}{y} = \frac{\sin(\pi/2)}{\pi/2} = \frac{1}{\pi/2} = \frac{2}{\pi}$

d. $\lim_{x\to 4^-} \frac{3-x}{x^2-2x-8} = \lim_{x\to 4^-} \frac{3-x}{(x-4)(x+2)} = +\infty$

negative

negative

form $\frac{c}{0}$

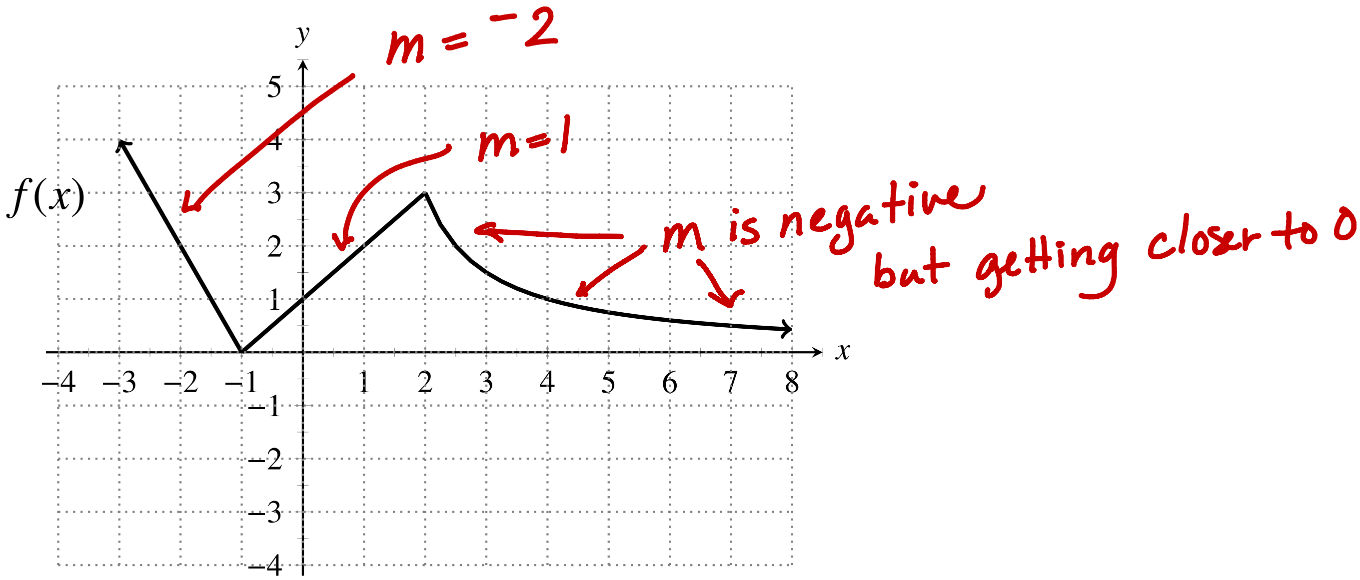
as $x\to 4^-$ (like $x\approx 3.9$)

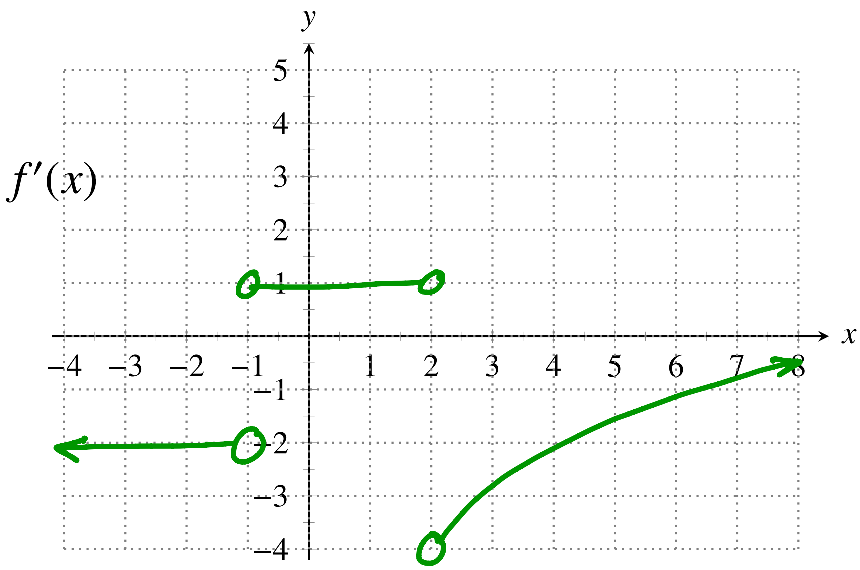
$3-x \to -1 \quad < 0$

$(x-4)(x+2) \to (0^-)(6) < 0$

3. (10 points)

Use the graph of $f(x)$ (on top) to sketch the graph of $f'(x)$ (on bottom).





4. (6 points)

Find an equation of the tangent line to the graph of $H(x) = 3 + \sqrt{x}$ at $x = 25$. You may use the fact that $H'(x) = \frac{1}{2\sqrt{x}}$.

$H(25) = 3 + \sqrt{25} = 8$

$H'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{10} = m$

$y - 8 = \frac{1}{10}(x - 25)$

So, $y = 8 + \frac{1}{10}(x-25)$

or

$y = \frac{1}{10}x + 5.5$

5. (15 points)

A drone is launched at time $t = 0$, and its height in meters at t seconds is

$$s(t) = 2 + \frac{10}{t+1}.$$

When answering the following questions, be sure to include **units** where appropriate.

a. From what height is the drone launched?

$S(0) = 2 + \frac{10}{0+1} = 12$ meters

The drone is launched from a height of 12 meters.

b. Compute the average velocity (i.e. the average rate of change of the height) from $t = 0$ to $t = 4$.

$V_{avg} = \frac{S(4) - S(0)}{4-0} = \frac{4-12}{4} = \frac{-8}{4} = -2$ m/sec

$S(4) = 2 + \frac{10}{4+1} = 2+2 = 4$

c. It is a fact that $s'(t) = \frac{-10}{(x+1)^2}$. Compute $s'(2)$ and explain what this calculation indicates about the drone.

$S'(2) = \frac{-10}{(2+1)^2} = \frac{-10}{9}$ m/s

The instantaneous velocity of the drone at 2 seconds is $\frac{-10}{9}$ m/s.

(OR) Exactly 2 seconds after launch, the height of drone is decreasing at a rate of $\frac{10}{9}$ m/s.

6. (10 points)

Consider the function

$$g(x) = \frac{1}{x+2}$$

Using the **definition of the derivative**, find $g'(a)$. No credit will be given if a different method is used. [It is recommended you start by writing the definition of the derivative.]

$$g'(a) = \lim_{h\to 0} \frac{g(a+h) - g(a)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{a+h+2} - \frac{1}{a+2}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{(a+2) - (a+h+2)}{(a+h+2)(a+2)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{-h}{(a+h+2)(a+2)}\right)$$

$$= \lim_{h\to 0} \frac{-1}{(a+h+2)(a+2)}$$

$$= \frac{-1}{(a+2)^2}$$

7. (15 points)

Consider the function $f(x) = \dfrac{2x-3}{x+1}$.

a. The graph of f contains a **horizontal** asymptote. Find it, and clearly state a specific limit that justifies this asymptote.

HA: $y=2$

Justification: $\lim\limits_{x\to\infty} \dfrac{2x-3}{x+1} = 2$

b. The graph $y = f(x)$ contains a **vertical** asymptote. Find it, and clearly state a specific limit that justifies this asymptote.

VA: $x=-1$

negative

Justification: $\lim\limits_{x\to -1^+} \dfrac{2x-3}{x+1} = -\infty$

positive

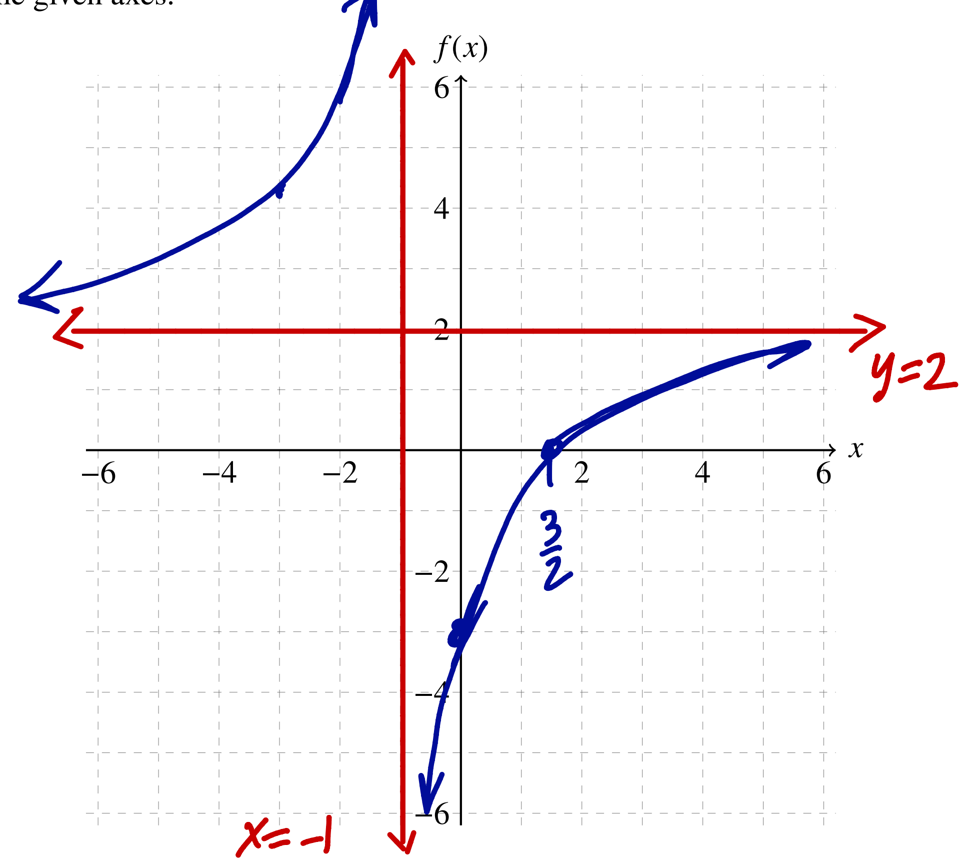
c. Sketch the graph of $f(x)$ on the given axes.

$x=0,\ y=-3$

$y=0,\ x=\frac{3}{2}$

$x=-2,\ y=\frac{-7}{-1}$

$x=-3,\ y=\frac{-9}{-2}$



8. (12 points)

Consider the function

$$f(x) = \frac{x-3}{x^2+2x-15}$$

a. What is the domain of f? Use interval notation.

$x^2+2x-15 = (x+5)(x-3) = 0$ means $x=3$ or $x=-5$

Domain: $(-\infty,-5)\cup(-5,3)\cup(3,\infty)$.

b. Compute: $\displaystyle\lim_{x\to 3}\frac{x-3}{x^2+2x-15} =$ $\displaystyle\lim_{x\to 3}\frac{x-3}{(x+5)(x-3)} = \lim_{x\to 3}\frac{1}{x+5} = \frac{1}{8}$

c. Explain why the given function is discontinuous at $a = 3$:

$$g(x) = \begin{cases} \dfrac{x-3}{x^2+2x-15} & \text{if } x \neq 3 \\ 4 & \text{if } x = 3 \end{cases}$$

$\displaystyle\lim_{x\to 3} g(x) = \frac{1}{8} \neq 4 = g(4)$

Extra Credit. **(3 points)**

Use the Intermediate Value Theorem to demonstrate that the two curves $f(x) = 4 - x^2$ and $g(x) = e^{2x}$ must intersect. To receive full credit you must state (i) **when** your are applying the Theorem and (ii) **how** you know the Theorem applies.

Let $H(x) = 4 - x^2 - e^{2x}$.

So f and g will intersect when $H(x) = 0$.

Observe that $H(0) = 3$ which is positive

and $H(10) = 4 - 100 - e^{200}$ which is definitely negative.

Since $H(x)$ is continuous, larger than zero at $x=0$, and smaller than zero at $x=100$, the Intermediate Value Theorem implies $H(x)$ must be equal to zero somewhere in the interval $(0, 100)$.