Math F251

Midterm 2

Spring 2020

Name: _____

Section: D F01 (Faudree) D F02 (Bueler) UX1 (Van Spronsen)

All students must affirm the following statements by initialing in the blanks provided. Students using their own paper must write out the statements in full.

_____ I will not seek or accept help from anyone.

I will not use a calculator, books, notes, the internet or other aids.

I understand that answers without work will not be awarded credit.

Good luck!

Problem	Possible	Score	
1	10		
2	10		
3	10		
4	8		
5	12		
6	6		
7	12		
8	10		
9	10		
10	12		
Total	100		

1. (10 points)

A table of values for f(x), g(x), f'(x) and g'(x) is given.

x	f(x)	f'(x)	g(x)	g'(x)
1	3	1	2	8
2	4	3	2	4
3	5	2	1	6

a. If $h(x) = x^2 f(x) - g(x)$, find h'(3).

b. If
$$h(x) = f(g(x))$$
, find $h'(1)$.

2. (10 points)

A particle moves on a vertical line so that its coordinate y at time t is $y = t^4 - 3t^2 + 2$, where $t \ge 0$.

a. What is the initial position of the particle?

b. When is the particle moving downward?

3. (10 points)

On March 21, the Alaska Department of Health and Social Services finds 21 Alaskans are infected with a new virus. By March 31, the number of Alaskans infected has risen to 133. Assume that the number of people infected grows at a rate proportional to the size of the infected population.

a. Write an equation that says that the number of people infected grows at a rate proportional to the size of the infected population.

b. Assuming the growth rate continues, with no mitigating factors, find an expression for the number, N, of Alaskans infected over time t in days.

4. (8 points)

Sketch a graph f with domain [1, 4] such that f has an absolute minimum but no absolute maximum.

5. (12 points)

A ship passes a lighthouse at 3:30pm, sailing to the east at 5 mph, while another ship sailing due south at 6 mph passes the same point half an hour later. How fast will the distance between the ships be increasing at 5:30pm?

6. (6 points)

Does the graph of the function $f(x) = \frac{3 \ln x}{1-x}$ have a vertical asymptote at x = 1? Justify your answer using an appropriate limit.

7. (12 points)

The graph of the *derivative* f' of a continuous function f is shown.



a. Determine the critical points of f(x).

b. At what values of x, does f have a local maximum? Local minimum? Explain your answer.

c. On what intervals is *f* concave upward? Concave downward? Use interval notation.

8. (10 points)

A function and its first and second derivatives are given below.

$$f(x) = x^{5/3} - 5x^{2/3}, \qquad f'(x) = \frac{5x - 10}{3x^{1/3}}, \qquad f''(x) = \frac{10x + 10}{9x^{4/3}}$$

a. Find the intervals of increase and decrease, and identify the locations of any local maximum or minimum values.

b. Find the intervals of concavity and the *x*-values of any inflection points.

9. (10 points)

Sketch a graph that satisfies all of the conditions:



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10. (12 points)

The graph of the function $f(x) = \sqrt{\frac{x}{2} + 1}$ is shown.



a. Let G(x) be the square of the distance from the origin to a point on the graph of y = f(x). Write an expression for G(x).

b. Use the expression for G(x) to find the closest point on the graph y = f(x) to the origin.

c. Show your result by adding a point, with coordinates, to the graph.