Name: $\qquad$

Section: $\square$ F01 (Faudree)
F02 (Bueler)
$\square$ UX1 (Van Spronsen)

All students must affirm the following statements by initialing in the blanks provided. Students using their own paper must write out the statements in full.
$\qquad$ I will not seek or accept help from anyone.
$\qquad$ I will not use a calculator, books, notes, the internet or other aids.
$\qquad$ I understand that answers without work will not be awarded credit.
Good luck!

| Problem | Possible | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 10 |  |
| 3 | 10 |  |
| 4 | 8 |  |
| 5 | 12 |  |
| 6 | 6 |  |
| 7 | 12 |  |
| 8 | 10 |  |
| 9 | 10 |  |
| 10 | 12 |  |
| Total | 100 |  |

1. (10 points)

A table of values for $f(x), g(x), f^{\prime}(x)$ and $g^{\prime}(x)$ is given.

| $x$ | $f(x)$ | $f^{\prime}(x)$ | $g(x)$ | $g^{\prime}(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 3 | 1 | 2 | 8 |
| 2 | 4 | 3 | 2 | 4 |
| 3 | 5 | 2 | 1 | 6 |

a. If $h(x)=x^{2} f(x)-g(x)$, find $h^{\prime}(3)$.
b. If $h(x)=f(g(x))$, find $h^{\prime}(1)$.

## 2. (10 points)

A particle moves on a vertical line so that its coordinate $y$ at time $t$ is $y=t^{4}-3 t^{2}+2$, where $t \geq 0$.
a. What is the initial position of the particle?
b. When is the particle moving downward?

## 3. (10 points)

On March 21, the Alaska Department of Health and Social Services finds 21 Alaskans are infected with a new virus. By March 31, the number of Alaskans infected has risen to 133. Assume that the number of people infected grows at a rate proportional to the size of the infected population.
a. Write an equation that says that the number of people infected grows at a rate proportional to the size of the infected population.
b. Assuming the growth rate continues, with no mitigating factors, find an expression for the number, $N$, of Alaskans infected over time $t$ in days.

## 4. (8 points)

Sketch a graph $f$ with domain $[1,4]$ such that $f$ has an absolute minimum but no absolute maximum.

## 5. (12 points)

A ship passes a lighthouse at $3: 30 \mathrm{pm}$, sailing to the east at 5 mph , while another ship sailing due south at 6 mph passes the same point half an hour later. How fast will the distance between the ships be increasing at $5: 30 \mathrm{pm}$ ?

## 6. (6 points)

Does the graph of the function $f(x)=\frac{3 \ln x}{1-x}$ have a vertical asymptote at $x=1$ ? Justify your answer using an appropriate limit.
7. (12 points)

The graph of the derivative $f^{\prime}$ of a continuous function $f$ is shown.

a. Determine the critical points of $f(x)$.
b. At what values of $x$, does $f$ have a local maximum? Local minimum? Explain your answer.
c. On what intervals is $f$ concave upward? Concave downward? Use interval notation.

## 8. (10 points)

A function and its first and second derivatives are given below.

$$
f(x)=x^{5 / 3}-5 x^{2 / 3}, \quad f^{\prime}(x)=\frac{5 x-10}{3 x^{1 / 3}}, \quad f^{\prime \prime}(x)=\frac{10 x+10}{9 x^{4 / 3}}
$$

a. Find the intervals of increase and decrease, and identify the locations of any local maximum or minimum values.
b. Find the intervals of concavity and the $x$-values of any inflection points.

## 9. (10 points)

Sketch a graph that satisfies all of the conditions:
domain $f=(-\infty, \infty)$,
$f(3)=-1, \quad f^{\prime}(3)=0$
$f^{\prime}(x)<0$ when $x<3, f^{\prime}(x)>0$ when $x>3$,
$f^{\prime \prime}(x)<0$ when $x<0, \quad f^{\prime \prime}(x)>0$ when $x>0$
4
$\lim _{x \rightarrow-\infty} f(x)=4$

10. (12 points)

The graph of the function $f(x)=\sqrt{\frac{x}{2}+1}$ is shown.

a. Let $G(x)$ be the square of the distance from the origin to a point on the graph of $y=f(x)$. Write an expression for $G(x)$.
b. Use the expression for $G(x)$ to find the closest point on the graph $y=f(x)$ to the origin.
c. Show your result by adding a point, with coordinates, to the graph.

