Math F251 Final Exam Spring 2022

Name: ______________________

Section: ☐ F01 (Faudree)
☐ F02 (Gossell)
☐ UX1 (Gossell)

Rules:

You have 2 hours to complete the exam.

Partial credit will be awarded, but you must show your work.

No other aids are permitted.

Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.

Turn off anything that might go beep during the exam.

Problem	Possible	Score
1	8	
2	8	
3	12	
4	8	
5	10	
6	8	
7	4	
8	10	
9	12	
10	10	
11	10	
Extra Credit	5	
Total	100	

1. (8 points)

Find the derivative of each of the following functions. You do not need to simplify your answer.

a. $f(x) = \sqrt{x}e^{2x}$

b. $g(x) = \left(\ln(x) + \frac{3x}{4}\right)^5$

2. (8 points)

Evaluate the definite integrals below. **Simplify** your final answers.

a. $\int_1^3 4x - 5\,dx$

b. $\int 3(\sin x)^4 \cos x\,dx$

3. (12 points)

Evaluate the following limits. You must show your work to earn full credit. If you apply L'Hopital's Rule, you should indicate this.

a. $\lim_{x \to 2} \dfrac{2x^2 - 8}{x^2 - 3x + 2}$

b. $\lim_{x \to 4} \dfrac{2 - \sqrt{x}}{4 - x}$

c. $\lim_{x \to \infty} \dfrac{45 + x - 4x^2}{x^2 - 16}$

4. (8 points)

a. Find the linear approximation (also known as the linearization) of the function $f(x) = \sqrt{x}$ when $a = 4$.

b. Use the linear approximation to estimate $\sqrt{4.04}$. Your answer must be in the form of a decimal.

5. (10 points)

(Optimization Problem) You need to construct a 100 ft^2 rectangular pen for a dog. Three sides of the pen (north, east, and south) will be made of open fencing which costs \$2 per foot. To add privacy, the west side which faces the street will be made of closed fencing which costs \$14 per foot. Follow the steps below to find the dimensions of the pen that minimize the cost.

a. Draw a diagram and label the sides.

b. Write an equation for the cost of the fencing in terms of a single variable.

c. Use Calculus to find the dimensions of the pen that **minimize** the cost. **Justify** your answer.

6. (8 points)

(Related Rates Problem) The volume V of a spherical snowball with radius r is given by the equation $V = \frac{4}{3}\pi r^3$. The surface area A is given by $A = 4\pi r^2$. Throughout the warm spring afternoon, the snowball melts at a constant rate of 36π cubic inches per hour.

a. At the moment that the radius is 6 inches, how fast is the radius decreasing? Include units in your answer.

b. At the moment that the radius is 6 inches, how fast is the surface area decreasing? Include units in your answer.

7. (4 points)

The number of subscribers to an internet streaming service is given by $s(t)$, where t is measured in months since the company started.

a. What does the statement $s'(36) = 4,580$ mean? Include units with your answer.

b. Would the owners of the streaming service prefer $s''(36)$ to be positive or negative? Explain your reasoning.

8. (10 points)

Suppose a particle moves along a straight line with **velocity** $v(t) = 3t^2 - 12t - 2$ m/s.

a. Find $s(t)$, the **position** of the particle at time t in seconds assuming that when $t = 1$ second the particle is at position $s = 10$ meters.

b. Find $a(t)$, the **acceleration** of the particle at time t in seconds.

c. At time $t = 0$, is the particle speeding up or slowing down? Explain your answer.

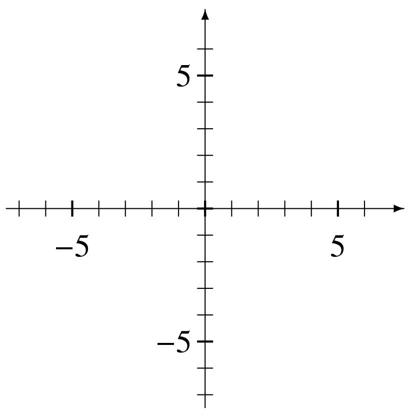
d. Determine the minimum velocity of the particle.

9. (12 points)

Sketch graphs which satisfy the given conditions. **There are many correct answers.**

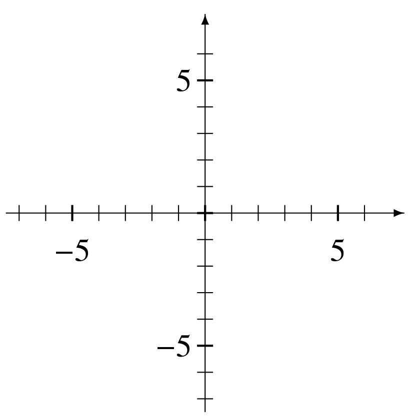
a. Sketch a graph of a function $f(x)$ that has

- an inflection point at $x = -5$, and
- a local minimum at $x = 5$.



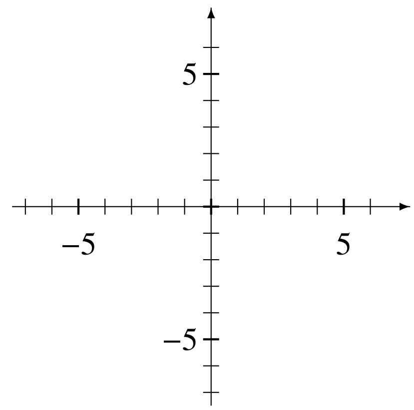
b. Sketch a graph of a function $g(x)$ that

- $\lim_{x \to 5^-} g(x) = 4$
- $\lim_{x \to 5^+} g(x) = -2$



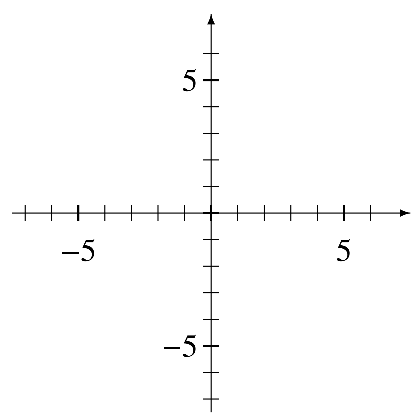
c. Sketch a graph of a function $h(x)$ such that

- $h'(-5) > 0$, and
- $h''(-5) < 0$.



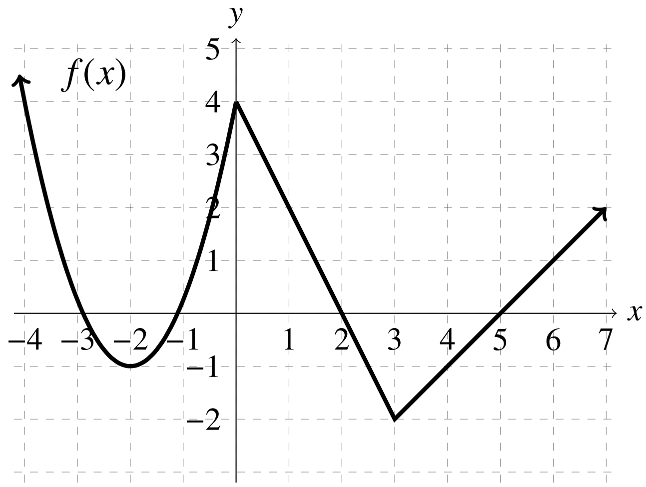
d. Sketch a graph of a function $k(x)$ such that

- $k'(-5) > 0$
- $k'(5) < 0$



10. (10 points)

Use the graph of the function $f(x)$ (on the right) to answer the questions below.



a. $\lim_{x\to 1} f(x) =$

b. $\lim_{x\to 1} \dfrac{f(1+h) - f(1)}{h} =$

c. At what values of x, if any, does the derivative, $f'(x)$, not exist?

d. On what intervals, if any, is $f'(x) > 0$?

e. Does $f(x)$ have any local minimums? If so, state the location and the local minimum value.

The following questions concern $G(x) = \int_0^x f(s)\, ds$.

f. What is the value of $G(4)$?

g. What is the value of $G'(4)$?

h. On the interval $[0, 7]$, does $G(x)$ have a local minimum? If so, state the location and the local minimum value.

11. (10 points)

A population of bacteria is growing at a rate of $p'(t) = 300e^{t/10}$ bacteria per day.

a. Compute $p'(0)$ and interpret its meaning in the context of the problem. Include units with your answer.

b. Compute $\int_0^{10} p'(t)\, dt$.

c. Interpret your answer from part (b) in the context of the problem. Make sure to include units.

12. (Extra Credit: 5 points)

Calculate $\dfrac{d}{dx}\left(\displaystyle\int_{\cos x}^{5} \frac{17^{-t}\ln(t+2)}{\sqrt{20-\sin^2 t}}\, dt\right)$.