

Name: _____**Section:** ☐ **001 (Jill Faudree)**
☐ **002 (James Gossell)**
☐ **005 (James Gossell)****Rules:**

You have 60 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

The exam is closed book and closed notes.

Calculators are not allowed.

Place a box around your **FINAL ANSWER** to each question where appropriate.

Turn off anything that might go beep during the exam.

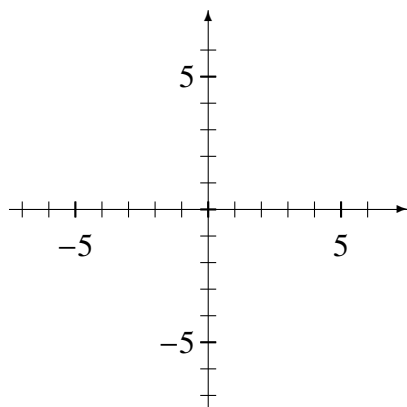
Good luck!

Problem	Possible	Score
1	12	
2	10	
3	10	
4	10	
5	10	
6	10	
7	16	
8	12	
9	10	
Extra Credit	5	
Total	100	

1. (12 points) Sketch graphs which satisfy the given conditions. **There are many correct answers.**

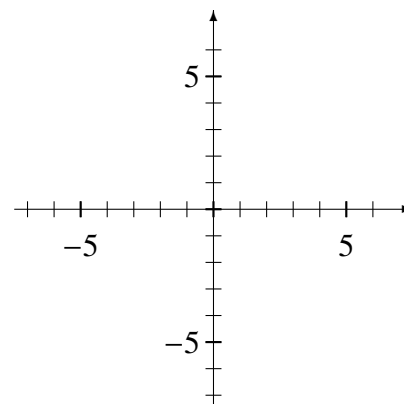
(a) Sketch a graph of a function $f(x)$ that has

- a local minimum at $x = -5$, and
- a local maximum at $x = 5$.



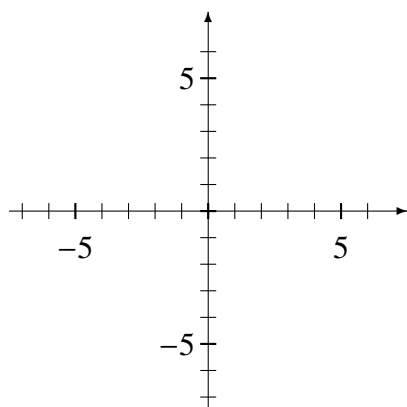
(c) Sketch a graph of a function $h(x)$ such that

- $h'(5) > 0$, and
- $h''(5) < 0$.



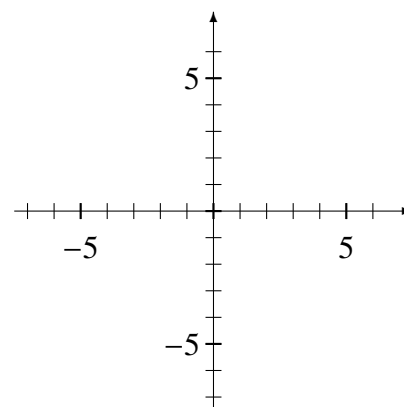
(b) Sketch a graph of a function $g(x)$ such that

- $\lim_{x \rightarrow \infty} g(x) = 3$, and
- $\lim_{x \rightarrow -\infty} g(x) = -3$.



(d) Sketch a graph of a function $k(x)$ that has

- $k''(x) > 0$ for $x < 5$, and
- $k''(x) < 0$ for $x > 5$.



2. (10 points) Evaluate the following limits. You must show your work to earn full credit. If you apply L'Hopital's Rule, you should indicate this.

(a) $\lim_{x \rightarrow 1} \frac{6x^2 - 12x + 6}{-x^4 + x^3 + x^2 - x}$

(b) $\lim_{x \rightarrow 0} \frac{x \cos 2x}{\sin 2x}$

3. (10 points)

- (a) Find the linear approximation (also known as the linearization) of the function $f(x) = x^3$ when $a = 2$.

- (b) Use the linear approximation to estimate $(2.02)^3$. Your answer must be in the form of a decimal.

4. (10 points) **[Related Rates Problem]** A hiker and a baby bear meet in the woods. Both are terrified! The hiker runs west at 3 meters per second. The bear lumbers south at 4 meters per second.

(a) Draw a diagram.

(b) Determine how fast the distance between the hiker and bear is increasing 2 seconds after the hiker and baby bear meet. Include units with your answer.

5. (10 points)

(a) Find all critical values of $f(x) = \frac{x^2 + 2}{2x + 1}$ on the interval $[0, 4]$.

(b) Identify the absolute maximum and absolute minimum of the function $f(x)$ on the interval $[0, 4]$.

Absolute max: $y =$

Absolute min: $y =$

6. (10 points) **[Optimization Problem]** Find the two **positive** numbers x and y on the circle $x^2 + y^2 = 48$ that maximize the product $P = xy^2$.

Note: Your solutions must use Calculus to **justify** that your answer is correct.

$x =$

$y =$

7. (16 points) Use the information below to answer questions about the function $f(x)$. You must show your work to earn full credit.

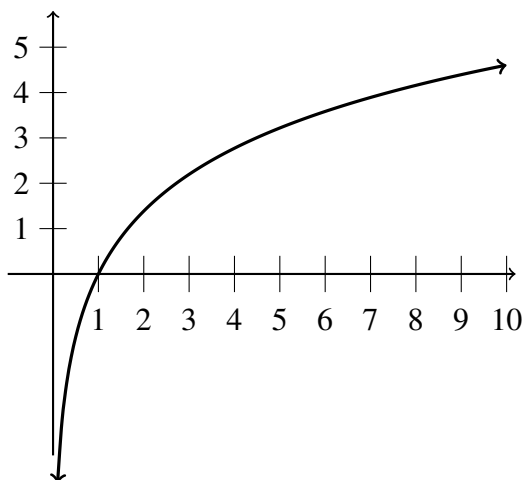
$$f(x) = x^{4/3} - 4x^{1/3}, \quad f'(x) = \frac{4x - 4}{3x^{2/3}}, \quad f''(x) = \frac{4x + 8}{9x^{5/3}}.$$

- (a) Determine the intervals on which $f(x)$ is increasing/decreasing.
- (b) Find the x -values that correspond to any local maximums or local minimums of $f(x)$.
- (c) Find the intervals on which $f(x)$ is concave up and concave down.
- (d) Find the x -values of any inflection points of $f(x)$. If there aren't any, you must explicitly state this and justify your answer.

8. (12 points)

- (a) Estimate the area under the curve $f(x) = \ln(x)$ on the interval $[1, 9]$ using R_4 . (That is, use 4 approximating rectangles and right-hand end points.) Sketch the approximating rectangles on the graph below.

Note: You are obviously not expected to compute things like $\ln(3)$. It is acceptable to have numbers like this in your final answer.



- (b) In fact, the area under the graph $f(x)$ on the interval $[1, 9]$ is about 11.8. Use this fact to evaluate the definite integrals below:

i. $\int_1^9 2 \ln(x) dx =$

ii. $\int_1^9 2 + \ln(x) dx =$

9. (10 points) Evaluate the indefinite integrals below.

(a) $\int (4 \cos(x) + x^{2.3} + 10) dx$

(b) $\int (x^2 + 4)^2 dx$

Extra Credit (5 points): The function $P(t) = \frac{5e^t}{e^t + 2}$ models a population of caribou (in hundreds) over time t in years. Evaluate the $\lim_{t \rightarrow \infty} P(t)$ and interpret the answer in the context of the problem. (Your interpretation should be a complete sentence that a regular person can understand.)