Math F251 Final Exam Spring 2023

Name: ______________________

Rules:

You have 2 hours to complete the final exam.

Partial credit will be awarded, but you must show your work.

You may have a single handwritten 3×5 notecard.

Calculators are not allowed.

Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	8	
2	8	
3	8	
4	10	
5	12	
6	10	
7	12	
8	12	
9	12	
10	8	
Extra Credit	5	
Total	100	

1. (8 points) Evaluate the limits. An answer without clear, mathematically precise work, will not earn full credit. Any use of L'Hôpital's Rule should be indicated using an **H** over the equal sign.

   (a) $\displaystyle\lim_{x\to 1} \frac{x^2-1}{\cos(\frac{\pi}{2}x)}$

   (b) $\displaystyle\lim_{x\to -\infty} \frac{1-x^3}{2x^3+4x^2-9}$

2. (8 points) Find the derivative. You do not need to simplify your answer.

   (a) $B(x) = (1+\sqrt{x})\ln(5x^2+x)$

   (b) $\cos(2x) + xe^y = 4y^3$ (Find $\dfrac{dy}{dx}$.)

3. (8 points) Evaluate the integrals. You do not need to simplify your answer.

   (a) $\displaystyle\int (3\sec^2(\theta) + \frac{6}{\theta} + \ln(2))\, d\theta$

   (b) $\displaystyle\int \frac{2}{1 + \frac{4}{9}t^2}\, dt$

4. (10 points) An box with a square base and an open top has volume 4000 cm^3. What dimensions of the box will minimize its surface area?
   You must show your work and use calculus to justify your answer.

   (a) Draw and label a diagram. Then write an equation for the surface area of the box in terms of a single variable.

   (b) Use Calculus to find the dimensions of the box that minimize the surface area.

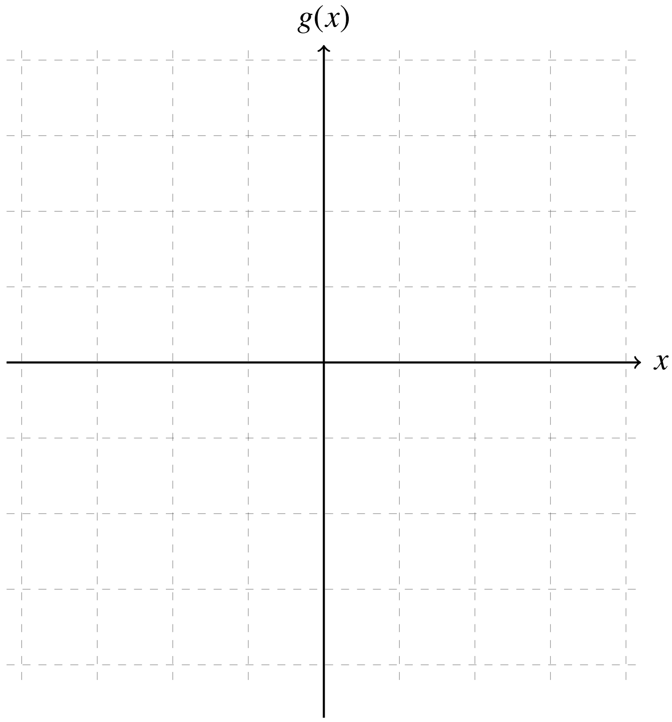
5. (12 points) The velocity of an object is given by $v(t) = \dfrac{t}{t^2+1}$ on the interval $[0, \infty)$, where v is measured in meters per second and t is measured in seconds.

   (a) Find an expression for $s(t)$, the position of the particle at time t, if $s = 1$ when $t = 0$.

   (b) Find an expression for $a(t)$, the acceleration of the object at time t.

   (c) Determine at what time, t, is the velocity of the particle maximized. Use Calculus to show that your answer is correct..

6. (10 points) Use the axes below to sketch a graph of a function $g(x)$ that satisfies **all** of the conditions in the bulleted list. Make sure to label any asymptotes, minimums or maximums, and inflection points. (See check list.)



- $g(x)$ is continuous on its domain $(-\infty, 1) \cup (1, \infty)$.
- $g(-1) = 1$, $g'(-1) = 0$
- $\lim_{x \to 1^-} g(x) = \infty$, $\lim_{x \to 1^+} g(x) = -\infty$
- $g'(x) > 0$ on $(-1, 1) \cup (1, \infty)$
- $g'(x) < 0$ on $(-\infty, -1)$
- $g''(x) > 0$ on $(-3, 1)$
- $g''(x) < 0$ on $(-\infty, -3) \cup (1, \infty)$

Did you ....
☐ label any asymptotes with its equation?
☐ label any maximums or minimums with **local min, local max, absolute min,** or **absolute max**?
☐ label any inflection points with **inflection point**?

7. (12 points) The **rate of change** of the volume of water in a tank is given by

$$r(t) = \frac{1}{2}t - 5$$

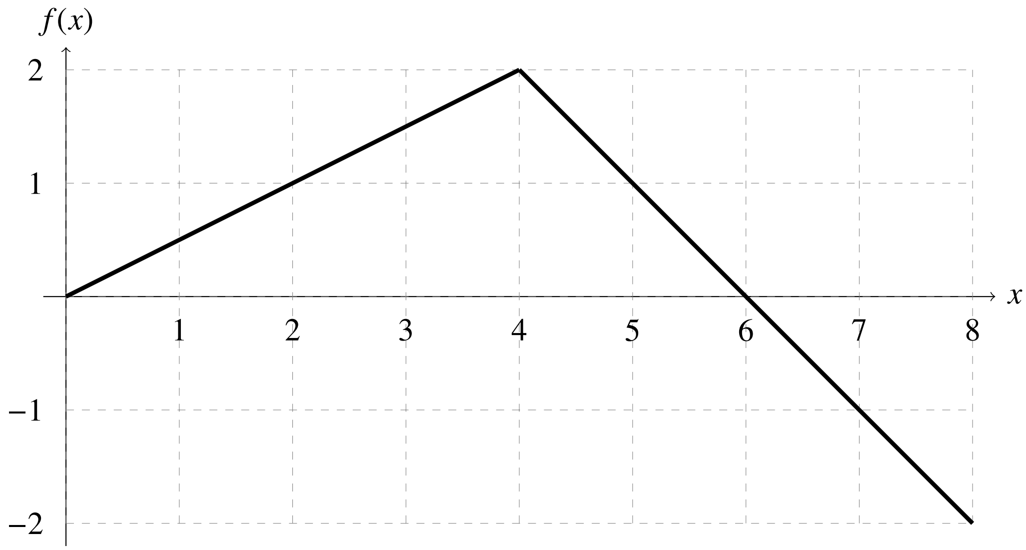
where r is measured in liters per minute and t is measured in minutes since the monitoring began.

(a) Compute $r(0)$ and $r(30)$. Then explain what these numbers mean in language the general public would understand.

(b) Compute the net change in the volume of water in liters from time $t = 0$ to time $t = 10$.

(c) At time $t = 0$, the tank contains 200 liters of water. What is the volume of water in the tank at time $t = 10$?

8. (12 points) Consider the function $f(x)$ with domain $[0, 8]$ graphed below.



(a) What is the value of $f(4)$?

(b) What is the value of $f'(2)$?

(c) Evaluate $\displaystyle\int_2^7 f(x)\,dx$.

The following questions concern $A(x) = \displaystyle\int_0^x f(s)\,ds$.

(d) What is the value of $A(4)$?

(e) What is the value of $A'(6)$.

(f) Does $A(x)$ have a maximum? Explain your answer.

9. (12 points) The temperature of a cup of coffee is modeled by the function

$$f(t) = 110e^{-t/10} + 40$$

where f is measured in degrees Fahrenheit and t is measured in minutes after the coffee was poured into the cup.

(a) Compute $f'(0)$. Then explain what this number means in language the general public could understand.

(b) Compute $\lim_{t\to\infty} f(t)$. Then explain what this number means in language the general public could understand.

10. (8 points) The radius of a spherical balloon is increasing at a rate of 2 cm/s. At what rate is the surface area of the balloon changing when the radius of the balloon is 5 cm? (Note that the surface area of a sphere is given by $SA = 4\pi r^2$.) Include units with your answer.

Extra Credit (5 points) The graph of the function $f(x) = \frac{1}{2}x^5 - x - \frac{1}{4}$ is shown.

a. Suppose Newton's method is used to find an approximate solution to $f(x) = 0$ from an initial guess of $x_1 = 1$. Sketch on the graph how the next approximation x_2 will be found, labeling its location on the x-axis.

b. For $x_1 = 1$, give a formula for x_2. You do not need to simplify, but your answer should be in a form where a calculator would compute a numerical value.