

Name: \_\_\_\_\_

**Rules:**

You have 90 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

You may have a single handwritten  $3 \times 5$  notecard.

Calculators are not allowed.

Place a box around your FINAL ANSWER to each question where appropriate.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	12	
2	20	
3	10	
4	8	
5	12	
6	15	
7	6	
8	17	
Extra Credit	5	
Total	100	

(a) State the definition of  $f'(x)$ , the derivative of the function  $f(x)$ .

(b) Find the derivative of  $f(x) = 4 - \sqrt{x}$  using the limit definition of the derivative. No credit will be awarded for using other methods.

2. (20 points) Evaluate the following limits. Show your work to earn full credit. Be careful to use proper notation.

(a)  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{2x^2 - 5x - 3} =$

(b)  $\lim_{x \rightarrow c} \frac{\frac{1}{c} - \frac{1}{x}}{x - c} =$

(c)  $\lim_{x \rightarrow 4^-} \frac{\sqrt{x}}{x^2 - 2} =$

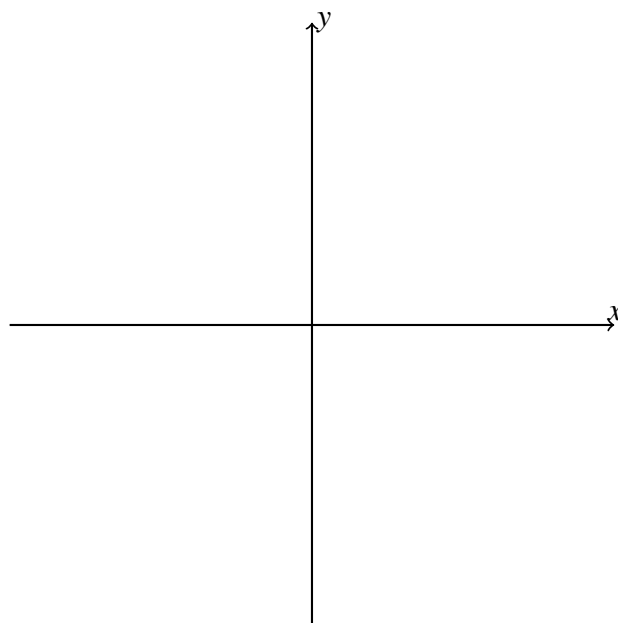
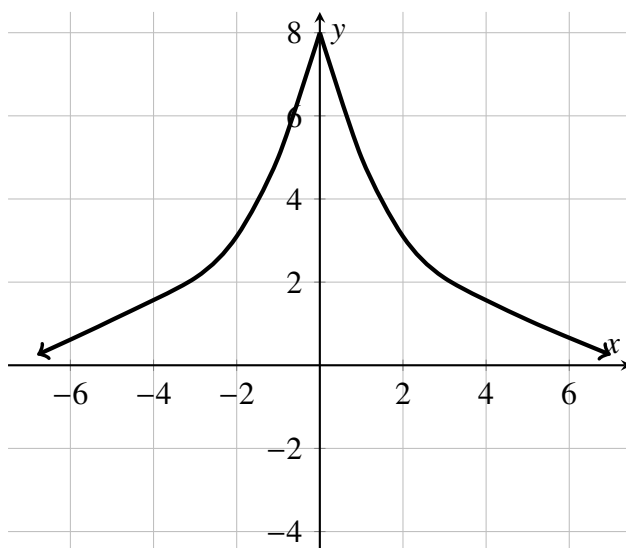
(d)  $\lim_{x \rightarrow -1^-} \frac{8 - 8x}{1 - x^2} =$

3. (10 points) Let  $f(x) = \begin{cases} \frac{\sqrt{x+5}}{2} & x < 9 \\ 6 & x = 9 \\ 3 + e^{x-9} & 9 < x \end{cases}$ .

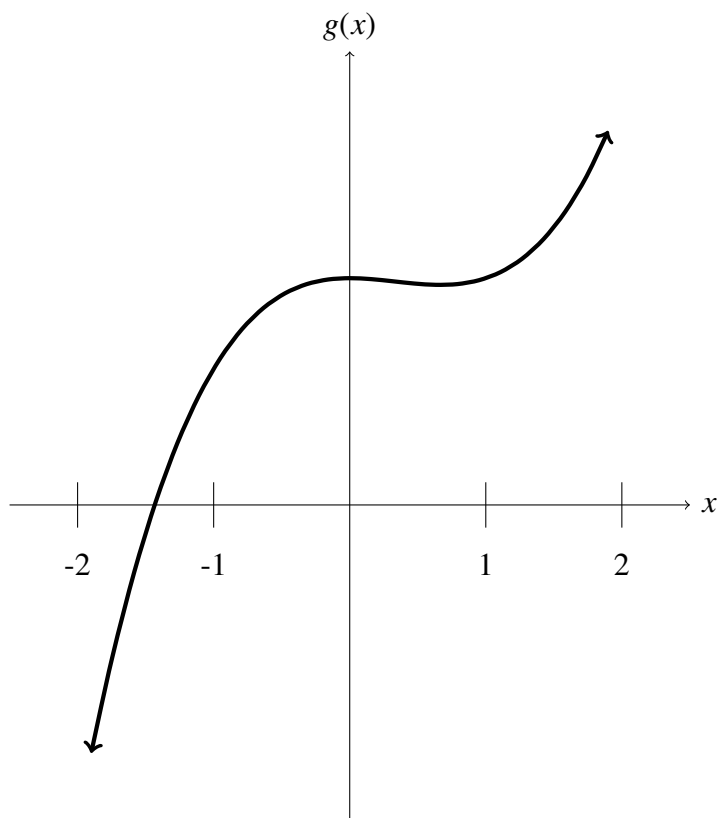
(a) Show that  $\lim_{x \rightarrow 9} f(x)$  exists.

(b) Determine if  $f(x)$  is continuous at  $x = 9$  and provide a mathematical justification for your conclusion.

4. (8 points) Use the graph of the function  $g(x)$  (drawn on the left below) to sketch the graph of  $g'(x)$  on the set of axes on the right.



5. (12 points) The function  $g(x) = x^2(x - 1) + 4$  is graphed below.



- (a) Sketch and label the **tangent** line to the graph of  $g(x)$  when  $x = -1$ .
- (b) Write an equation of the tangent line to  $g(x)$  when  $x = -1$ .
- (c) Find all  $x$ -values where the graph of  $g(x)$  has a horizontal tangent or explain why none exist.

6. (15 points) Find the derivative of each function below. You do not need to simplify your answer.

(a)  $f(x) = 2x^5 + \frac{2}{x^3} + 2\sqrt{5}$

(b)  $H(\theta) = \theta^{2/3} \sin(\theta)$

(c)  $j(x) = \frac{x}{\pi + \cos(x)}$

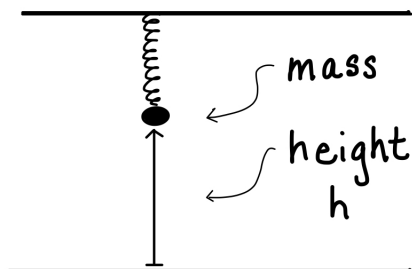
7. (6 points) The function  $V(T)$  models the volume of a fixed mass of gas with respect to temperature where volume  $V$  is measured in milliliters (or  $mL$ ) and temperature  $T$  is measured in kelvins (or  $K$ ).

(a) Interpret in the context of the problem the meaning of  $V(100) = 60$ . Include units.

(b) Interpret in the context of the problem the meaning of  $V'(100) = 0.4$ . Include units.

(c) Using the facts that  $V(100) = 60$  and  $V'(100) = 0.4$ , estimate  $V(110)$ . Include units.

8. (17 points) A spring is hanging from the ceiling with a mass attached to it. The mass is oscillating vertically with simple harmonic motion. The function  $h(t) = 6 - \cos(t)$  models the height of the spring above the floor starting at time  $t = 0$  where  $h$  is measured in feet and  $t$  is measured in seconds. **For all problems below, include appropriate units.**



- (a) Find the average velocity of the mass in the time interval  $[0, \pi]$  seconds.
- (b) Find the equations for the velocity and acceleration of the mass.
- (c) Find the instantaneous velocity of the mass at  $\pi/4$  seconds.
- (d) At  $t = \pi/4$ , is the mass going up or going down? Justify your answer.
- (e) Determine all times  $t > 0$  at which the mass changes direction?

**Extra Credit:** (5 points) Use the Intermediate Value Theorem to demonstrate that the function  $f(x) = x^3 - 3x - 19$  has a root (or zero) in the interval  $[1, 5]$ . A complete answer requires some calculations **and** complete sentences justifying your conclusion.