Spring 2024   Math F251X

Calculus 1: Midterm 2

Name: _____   **Section:** ☐ **9:15am (James Gossell)**
                                              ☐ **11:45am (Mohamed Nouh)**
   ☐ **async (Leah Berman)**

Rules:

- Partial credit may be awarded, but you must show your work.

- You may have a single handwritten $3'' \times 5''$ notecard, both sides.

- Calculators are **not** allowed.

- Place a box around your FINAL ANSWER to each question where appropriate.

- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	12	
2	6	
3	7	
4	10	
5	18	
6	11	
7	12	
8	12	
9	12	
Extra Credit	5	
Total	100	

1. (12 points)

A rectangular storage container is to be constructed to have a fixed volume of 8 m^3. Its bottom and sides are to be made of material that costs \$1 per square meter, while the top is constructed of material that costs \$5/square meter. The length of the base of the box is twice the width.
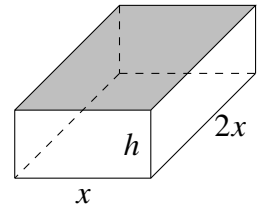
a. Write h in terms of x using the information in the problem.

b. Write a function for the **total cost of the box** of the box in terms of the variable x.

c. Determine the **dimensions** of the box with minimum cost.

Show your work, and use calculus to **justify** that your answer is the minimum. Include units in your final answer. An answer with no clear justification or that does not use calculus techniques will not receive full credit.

Dimensions: width: _____ length: _____ height: _____

2. (6 points)

Suppose the side length of a cube is measured to be 5 cm, with a possible measurement error of $\pm\frac{1}{10}$ cm.

a. Use linearization or differentials to estimate the possible error in the volume of the cube.

b. What is the relative error in the volume?

3. (7 points)

Find the absolute maximum and absolute minimum for the function

$$g(x) = (x - 2)(x + 3)^2 = x^3 + 4x^2 - 3x - 18$$

on the interval $[-4, 0]$. Justify how you know that you have found the absolute maximum and minimum. If there is no absolute maximum or minimum write "none".

Absolute maximum: $y =$ _____   Absoute minimum: $y =$ _____

4. (10 points)

A small airplane is flying horizontally at an altitude of 3000 m at a speed of 450 km/hr = 125 m/s.

At time $t = 0$ it passes above a camera on the ground that is pointed directly up. To keep the airplane in view, the angle of the camera changes. How fast is the angle θ (see diagram) changing when the plane has travelled 4000 meters from the spot directly above the camera? Give your answer as an exact number, and include units.
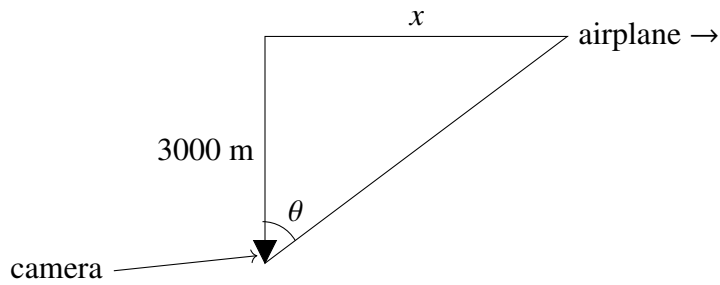
5. (18 points)

Answer the questions below about the function $f(x) = \dfrac{45x^4 - 1584x^2}{440\sqrt[3]{x}}$. It is a fact that after simplification,

$$f'(x) = \frac{3x^3 - 48x}{8\sqrt[3]{x}}, \quad \text{and } f''(x) = \frac{x^2 - 4}{\sqrt[3]{x}}$$

You must show your work and justify your conclusion with a few words or a computation. Make sure someone else can follow your work.

a. What is the domain of $f(x)$? (use interval notation) _____

b. Determine the intervals where f is **increasing** and where f is **decreasing**. Show your work.

Increasing: _____ Decreasing:_____(If none write "none".)

c. Fill in the blanks: $f(x)$ has a local maximum at $x =$ _____ and a local minimum at $x =$ _____. (If none, write "none".)

d. Find all intervals where f is **concave up** and where f is **concave down**. Show your work.

Concave up: _____Concave down:_____(If none write "none".)

e. Fill in the blanks: $f(x)$ has (an) inflection point(s) at $x =$ _____. (If none, write "none".)

6. (11 points)

Sketch a graph of a function $h(x)$ that satisfies all of the following properties.

After drawing the graph:

- **Label** on the graph the following things, if they exist, by drawing a point on the graph and labeling: any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP

- Draw any horizontal and vertical asymptotes with dashed lines and **label** them with their equation.

- Mark any important x-values and y-values on the x- and y-axes.

Properties:

- The domain of $h(x)$ is $(-\infty, \infty)$.

- $h(0) = 2$.

- $h'(x) > 0$ when $x < 0$.

- $h'(x) < 0$ when $x > 0$.

- $h''(x) > 0$ when $x < -2$ or $x > 2$.

- $h''(x) < 0$ when $-2 < x < 2$

- $\lim\limits_{x \to -\infty} h(x) = -2$

- $\lim\limits_{x \to \infty} h(x) = 1$
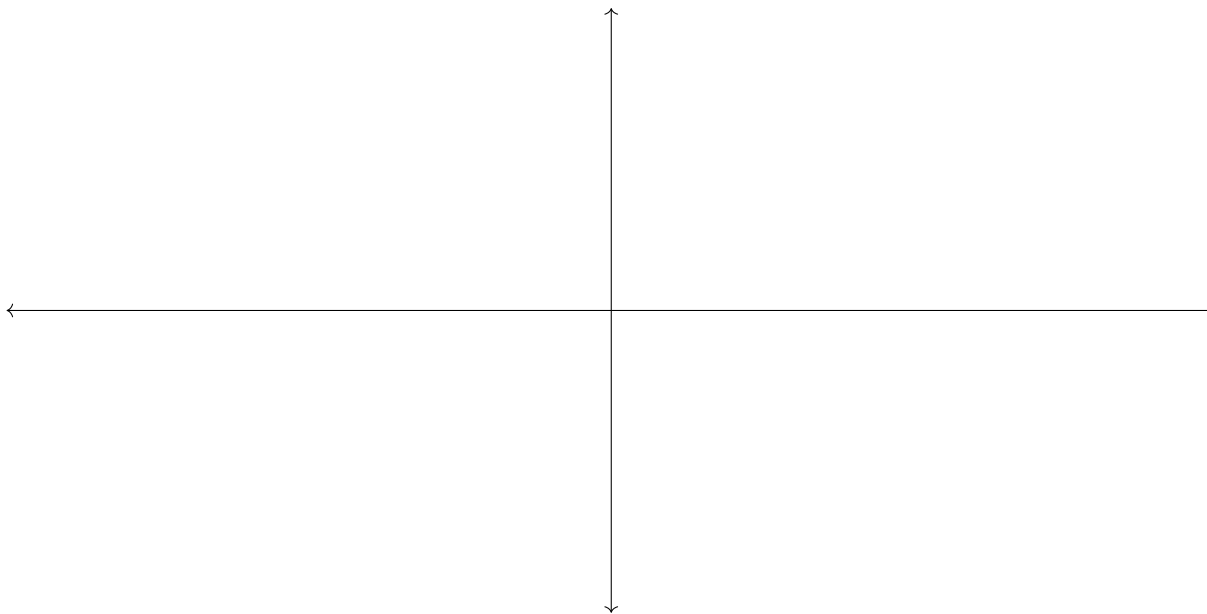
7. (12 points)

Evaluate the following limits. **Show your work**, uncluding appropriate use of limit notation. If you use L'Hôpital's rule, you must indicate where you are using it by writing $\overset{H}{=}$ or $\overset{L'H}{=}$ or something similar. Use ∞ or $-\infty$ where appropriate, and if the limit does not exist, write DNE and provide a justification.

a. $\lim\limits_{x \to 4} \dfrac{\ln(x-3)}{\sqrt{x}-2}$

b. $\lim\limits_{x \to 0} \dfrac{x^3 - x^2}{3\cos x - 3}$

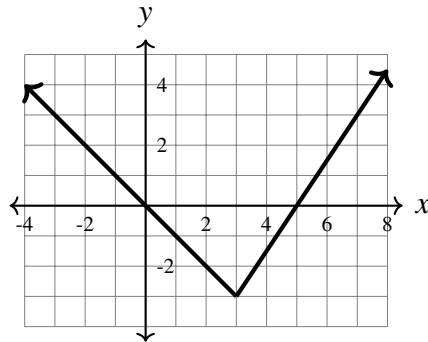**c.** $\lim\limits_{x \to \infty} \dfrac{e^{-x} + 2}{\sec\left(\frac{1}{x}\right)}$

d. $\lim\limits_{x \to \infty} \dfrac{12x^{10} + 8x^2 - 2}{7x^9 + 11x^4 + 6}$

8. (12 points)

A portion of the piecewise-linear function

$$f(x) = \begin{cases} -x & x \leq 3 \\ \frac{3}{2}(x-5) & x > 3 \end{cases}$$

is graphed below.

a. We want to approximate $\int_{-2}^{7} f(x)\, dx$ using three left-hand rectangles.

   i. **Draw** the three **left-hand** rectangles on the graph. Lightly shade them in. Make sure I can tell where your rectangles are and try to be reasonably precise.

   ii. Now **approximate** $\int_{-2}^{7} f(x)\, dx$ using the three **left-hand** rectangles. (Your answer should be a number.) Show some work.

b. Use geometry to compute $\int_{-2}^{7} f(x)\, dx$ exactly. Show your work.
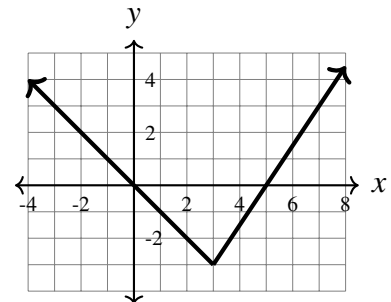
9. (12 points)

Answer the following, showing work where necessary.

a. $\displaystyle\int 4x - (\sec x)^2 \, dx$ (give the most generic answer)

b. $\displaystyle\int \frac{x^4 - 3x^2 + 5}{x^4} \, dx$ (give the most generic answer)

c. The **velocity** $v(t)$ of a particle is given by the function

$$v(t) = \sin(t) - \cos(t)$$

and it has a position function $s(t)$. At time $t = 0$, $s(0) = 1$. Determine the position function $s(t)$.

Extra Credit (5 points) The growth multiplier over a year for a savings account with an interest rate of 10% compounded x times per year is given by

$$G(x) = \left(1 + \frac{0.1}{x}\right)^x.$$

For example, $G(1) = 1.1$ means that you will multiply your money by 1.1 if interest is compounded once per year. $G(2) = 1.1025$ means that you will multiply your money by 1.1025 if interest is compounded twice per year.

Calculate $\lim\limits_{x \to \infty} G(x)$. What does this value mean in the context of the problem?