Spring 2025 **Math F251X**

Calculus 1: Midterm 1

Name: _____ **Section:** ☐ **9:15am (James Gossell)**
☐ **11:45am (Kevin Meek)**
☐ **async (Deven Barnett)**

Rules:

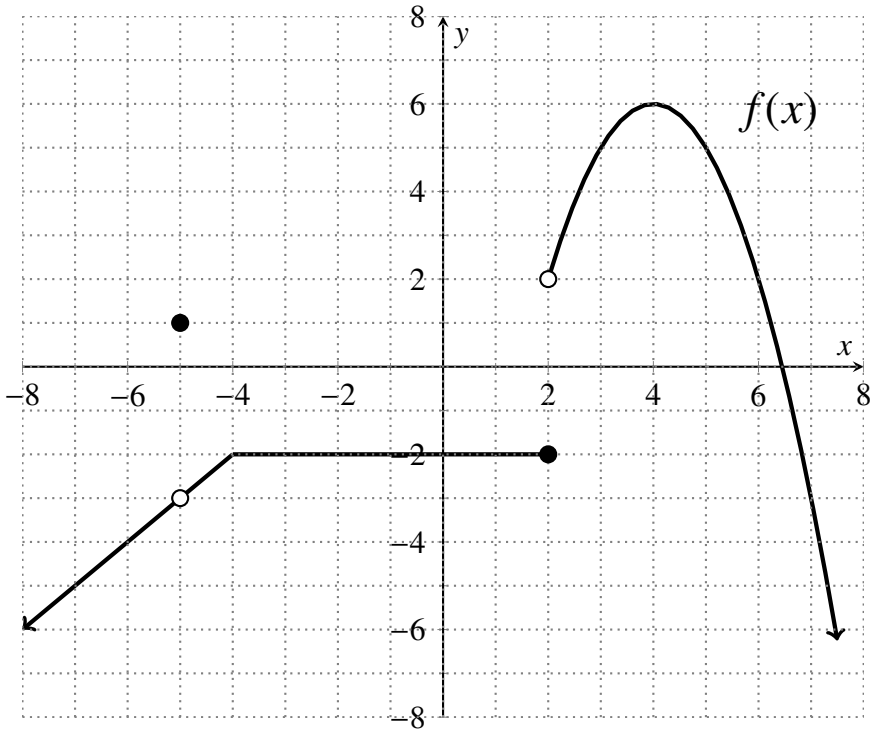
- Partial credit will be awarded, but you must show your work.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Calculators are **not** allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	12	
2	12	
3	6	
4	6	
5	10	
6	8	
7	6	
8	12	
9	12	
10	16	
Extra Credit	(5)	
Total	100	

1. (12 points)

Use the graph of $f(x)$ to answer the following questions.



a. Fill in the blanks below. If the value does not exist or is undefined, write DNE.

$\lim_{x\to -5} f(x) =$ _____ $f(-5) =$ _____ $\lim_{x\to -4} f(x) =$ _____

$\lim_{x\to 2^+} f(x) =$ _____ $f(2) =$ _____ $\lim_{x\to 2} f(x) =$ _____

$f'(-6) =$ _____ $f'(4) =$ _____ $\lim_{x\to -4^+} f'(x) =$ _____

b. State the x-values for which f is **not continuous**.

c. State at least one x-value for which f is **not differentiable** (where $f'(x)$ does not exist).

2. (12 points)

Compute the following limits. Show your work and use limit notation where necessary. You will be graded both on your computation and your correct use of notation.

a. $\lim_{x \to 3} \dfrac{x^2 - 5x + 6}{3 - x}$

b. $\lim_{x \to 1} \dfrac{1 - \dfrac{1}{x^2}}{x - 1}$

c. $\lim_{x \to 4} \dfrac{\sqrt{x} + 2}{x + 4}$

3. (6 points)

Determine whether or not the given function is continuous at $x = 3$. **Justify your answer using limits**.

$$f(x) = \begin{cases} x^2 - 4 & \text{if } x > 3 \\ 2x + 1 & \text{if } x \leq 3 \end{cases}$$

4. (6 points)

Find the value(s) of k that make the function continuous at $x = -2$.

$$g(x) = \begin{cases} \frac{x^2+3x+2}{x+2} & \text{if } x \neq -2 \\ k & \text{if } x = -2 \end{cases}$$

5. (10 points)

Use the limit definition (given below) of the derivative to find the derivative of

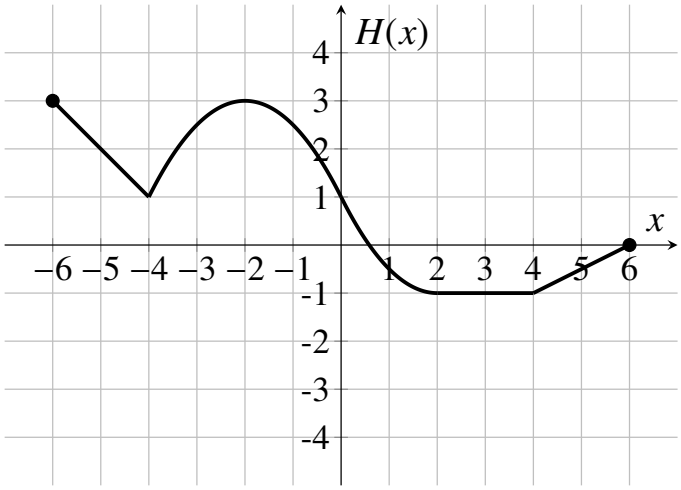
$$f(x) = 3x^2 - 4x.$$

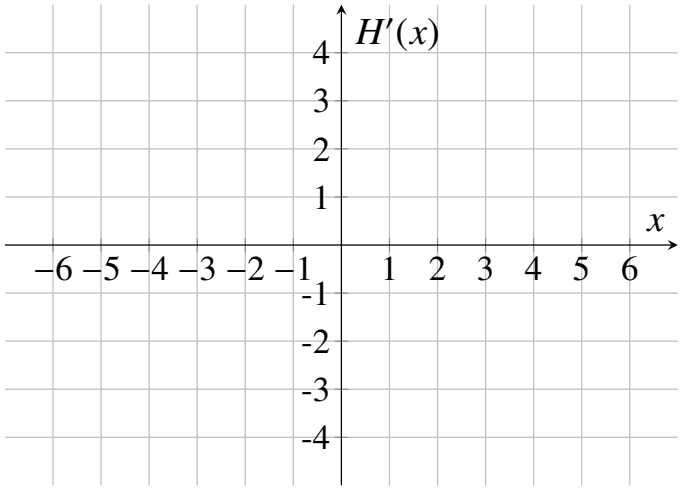
Show all your work clearly, step by step, using correct notation. **No credit will be awarded for a solution that does not use the definition below.**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

6. (8 points)

The function $y = H(x)$ is graphed below. Sketch the graph of $H'(x)$ on the blank set of axes provided.





7. (6 points)

$S(t)$ is a function that describes the human population of Lewisburg, West Virginia t years after 1970 when the population was first recorded.

a. Interpret the meaning of $\boxed{S(10) = 2570}$ in the context of the problem. Write a sentence or two including appropriate units.

b. Interpret the meaning of $\boxed{S'(10) = -135}$ in the context of the problem. Write a sentence or two including appropriate units.

c. Given $S(10) = 2570$ and $S'(10) = -135$, **estimate** $S(9)$ including units. Write a sentence explaning how you arrived at your estimation.

8. (12 points)

For each of the following functions, compute the derivative. **You do not need to simplify your answers.** Your answer must begin with $f'(x)$ or similar notation, as appropriate to the problem.

a. $f(x) = \dfrac{x^3 + 3x^2 + 2}{\sqrt{x}}$

b. $g(y) = y^{-6}\cos(y) + 5^2 - 4\pi y$

c. $g(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$

9. (12 points)

Consider the function $g(x) = \dfrac{1}{x^2} + \dfrac{x^4}{4}$.

a. Find $g'(x)$. (Use differentiation rules, not the definition of the derivative.)

b. Determine the x-values for which the function has a horizontal tangent line.

c. Is the function increasing or decreasing at $x = -2$? Show your work.

d. Find the equation of the tangent line to $g(x)$ at $x = -2$.

10. (16 points)

The position of a lady bug on a branch is given by $P(t) = t^3 - \frac{11}{2}t^2 + 6t + 8$ where time t is in minutes between 0 and 20 and position $P(t)$ is in inches.

a. Find the function describing the **velocity** of the lady bug.

b. At what times did the lady bug stop moving?

c. $P(t)$ is measured from the trunk of the tree meaning $P(t) = 0$ indicates that the lady bug is at the trunk, the base of the branch. At time $t = 2$ minutes, is the lady bug moving away from the trunk or towards the trunk? Show your work.

d. At time $t = 2$ minutes, is the lady bug speeding up or slowing down? Explain.

11. (Extra Credit: 5 points)

Find a point on the graph of $f(x) = x^2 + x - 1$ such that the tangent line at that point has a y intercept at $(0, -2)$.