

Math F251X Calculus 1

Introduction to Section 1.1: functions and models

August 24, 2020

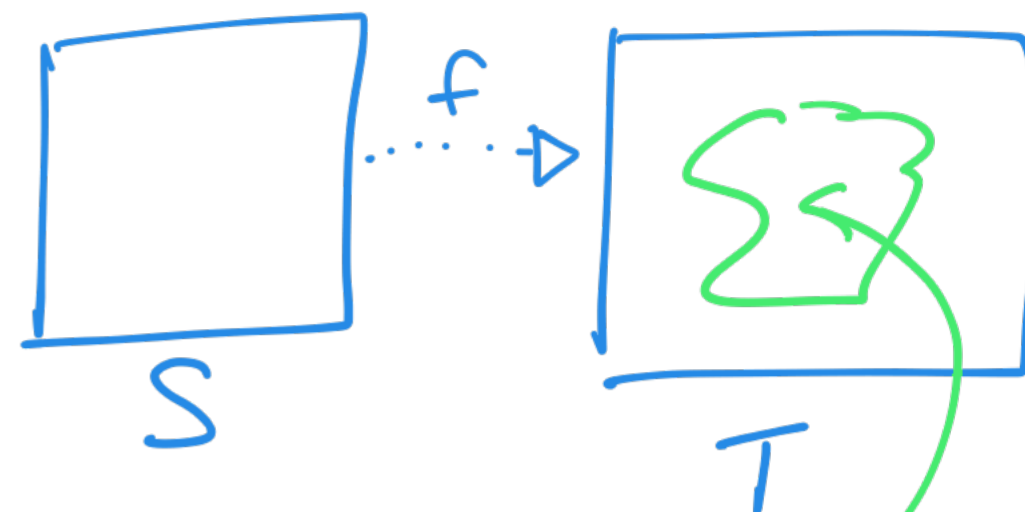
What is a function?

A rule

$$f: S \rightarrow T$$

↑
input space
DOMAIN

output space
CODOMAIN



Stuff actually
hit = RANGE

• As a list: $f = \{1, 2, 3\} \rightarrow \{4, 5, 6\}$

by $1 \mapsto 4$

$2 \mapsto 6$

$3 \mapsto 6$

$f(1) = 4$

or $f(2) = 6$

$f(3) = 6$

range = $\{4, 6\}$

In calculus: specify functions as a rule:

$$f: \mathbb{R} \rightarrow \mathbb{R} \quad \text{by} \quad f(x) = x^2.$$

To be a function:

- ① every input must have some output.
- ② No input can produce multiple outputs.

Graph of a function:

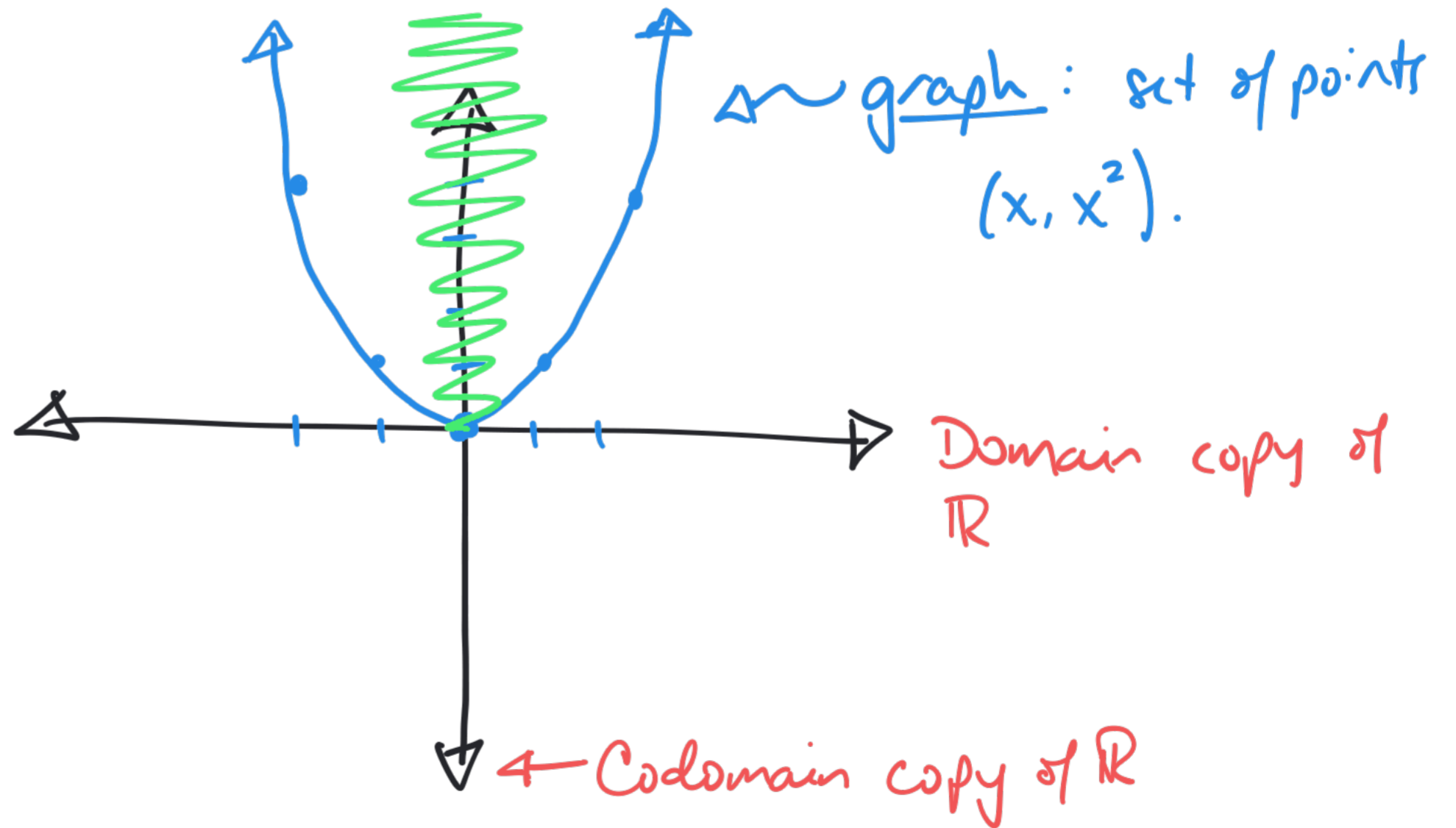
given $f: \mathbb{R} \rightarrow \mathbb{R}$, we plot ordered pairs $(x, f(x))$

Ex: $f(x) = x^2$

Range:

all real #s ≥ 0

$[0, \infty)$



Functions and rules

Example: $f(\square) = 3(\square)^2 - 5$.

What is:

① $f(2)$? $f(2) = 3(2)^2 - 5$
 $= 3 \cdot 4 - 5 = 12 - 5 = 7$.

② $f(a)$? $f(a) = 3a^2 - 5$.

③ $f(y^2)$? $f(y^2) = 3(y^2)^2 - 5 = 3y^4 - 5$

What about $(f(y))^2$?

$$(f(y))^2 = (3y^2 - 5)^2 = (3y^2)^2 - 2(3y^2)(-5) + (-5)^2$$
$$= 9y^4 + 30y^2 + 25$$

④ $f(a+h)$?

$$f(a+h) = 3(a+h)^2 - 5 = 3(a^2 + 2ah + h^2) - 5$$
$$= 3a^2 + 6ah + h^2 - 5$$

Special Kinds of Functions

① polynomials

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

a_i are coefficients; a_0 is the constant term

Example: $f(x) = x^2 = 1x^2 + 0$ ← degree 2 = quadratic

degree 1 = linear

degree 3 = cubic

② linear functions / lines

$$f(x) = y = a_1 x + a_0$$



$$y = mx + b$$

↑
slope



"y-intercept"

• Where the function intersects the y-axis!

point-slope form:

given a point (x_1, y_1) on line

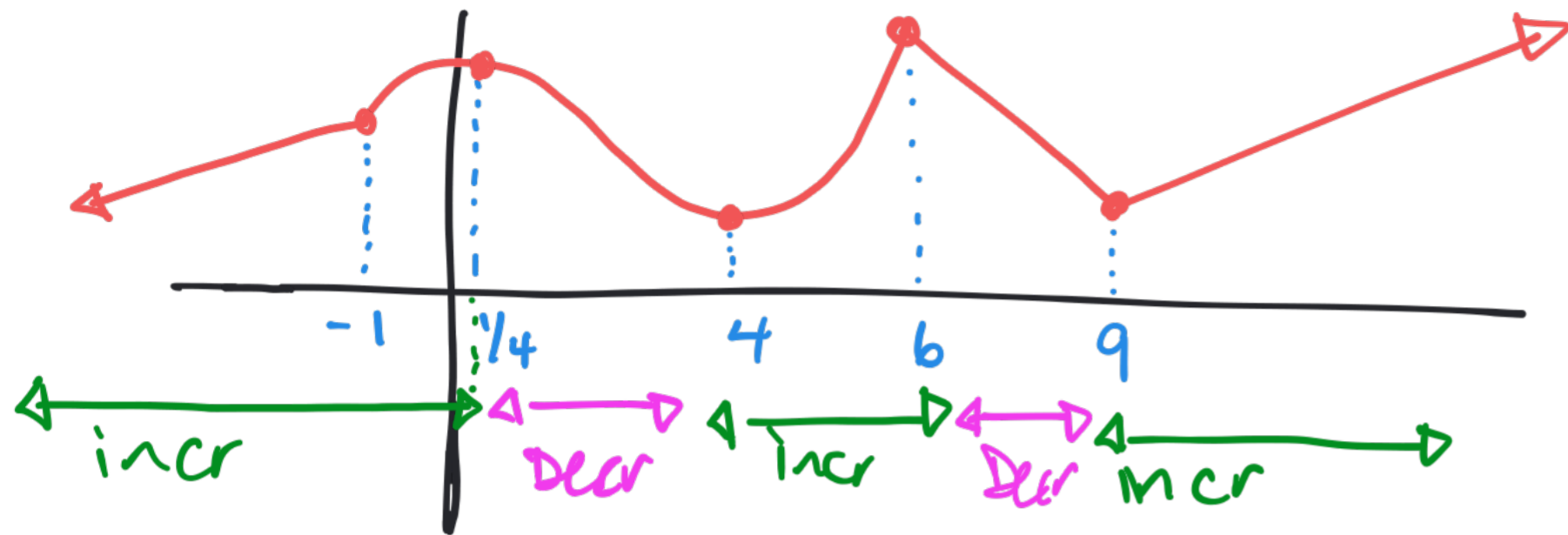
and slope m :

equation is

$$\boxed{y - y_1 = m(x - x_1)} \Rightarrow y = m(x - x_1) + y_1$$

If $f(x) = mx + b$ then $f(0) = b$ ↑

③ Increasing & decreasing



Increasing:

if $x_1 < x_2$ then $f(x_1) \leq f(x_2)$
 as x -values increase
 y -values increase

Decreasing:

if $x_1 < x_2$ then $f(x_1) \geq f(x_2)$

④ Piecewise

Example: $f(x) = \begin{cases} x & \text{if } x < 2 \\ \frac{1}{2}x + 3 & \text{if } x \geq 2 \end{cases}$

What's the range?

$(-\infty, 2) \cup [4, \infty)$

