Intro video: Section 1.4
Exponential Functions

Math F251X: Calculus 1

Recall the laws of exponents

① $a^m b^m = (ab)^m$

$$\underbrace{(aa\cdots a)}_{m}\underbrace{(bb\cdots b)}_{m} = \underbrace{(ab)\cdots(ab)}_{m}$$

② $a^m a^n = a^{m+n}$

$$\underbrace{(a\cdot a\cdots a)}_{m}\underbrace{(a\cdots a)}_{n} = \underbrace{a\cdots a}_{m+n}$$

③ $(a^m)^n = a^{mn}$

$$(a^m)^n = \underbrace{a^m\cdots a^m}_{n} = a^{\overbrace{m+m+\cdots+m}^{n}} = a^{nm}$$

④ $a^{1/n} = \sqrt[n]{a}$

⑤ $a^{-1} = \frac{1}{a}$

Example: Write the following with no negative exponents as simply as possible

$$\frac{(ga)^2 y^{-3} t^5}{\frac{1}{2}\sqrt{t}\cdot g^{-4}} = \frac{(ga)^2 t^5 \cdot g^4}{\frac{1}{2} t^{1/2} y^3} = \frac{g^2 a^2 t^5 g^4}{1/2\, t^{1/2} y^3}$$

$$= \frac{g^2 a^2 t^5 t^{-1/2} g^4 \cdot 2}{y^3} = \frac{(g^2 g^4)(t^5 t^{-1/2})\cdot 2a^2}{y^3}$$

$$= \frac{(g^{2+4})(t^{5-1/2})\cdot 2a^2}{y^3} = \frac{g^6 t^{9/2}\cdot 2a^2}{y^3} = \frac{2g^6 a^2 t^4\sqrt{t}}{y^3}$$

Exponential Functions

$f(x) = C\, b^{x}$

Value at $x=0$

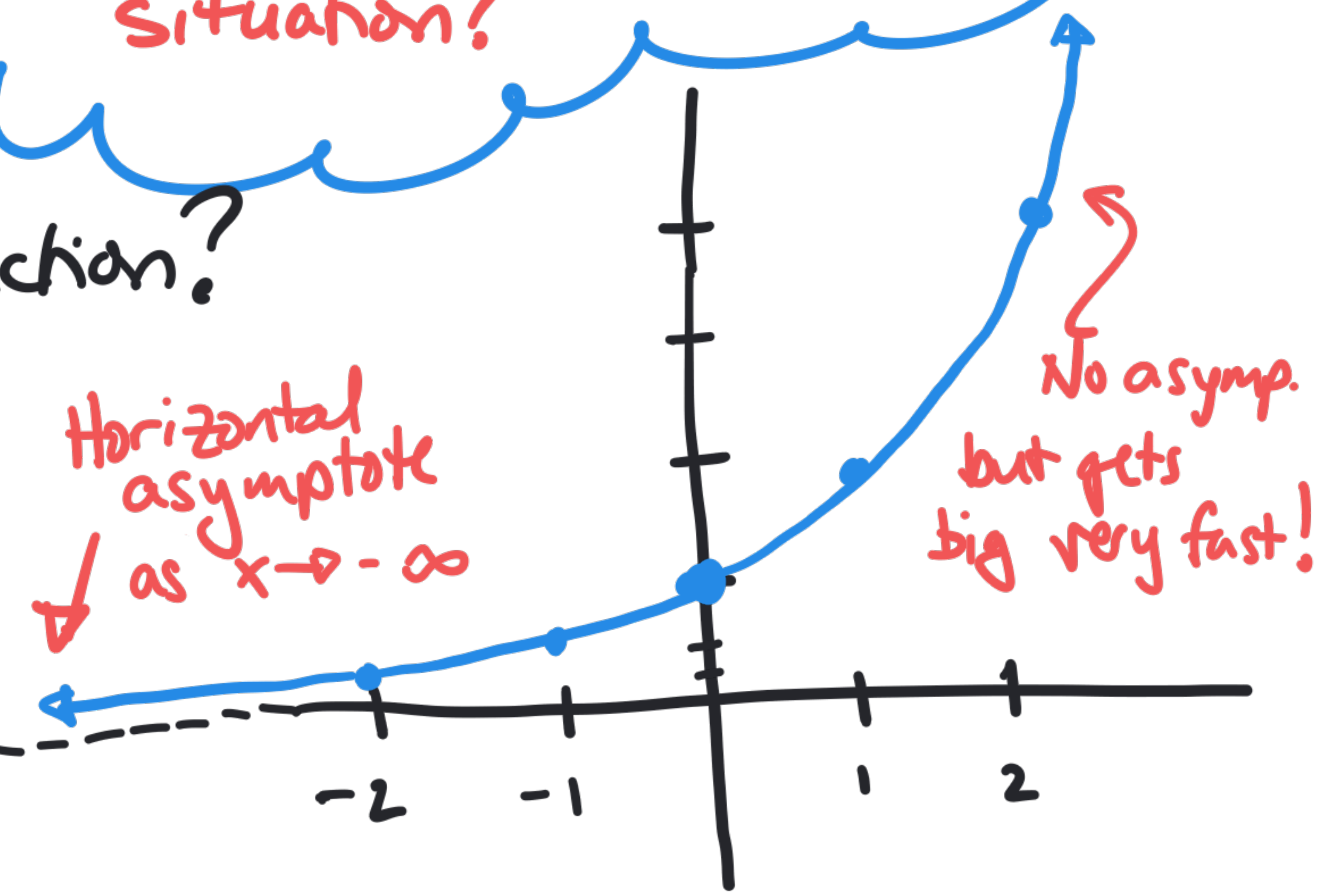
base

A population starts with 3 infected people. Every day, each infected person infects one new person. What function models this situation?

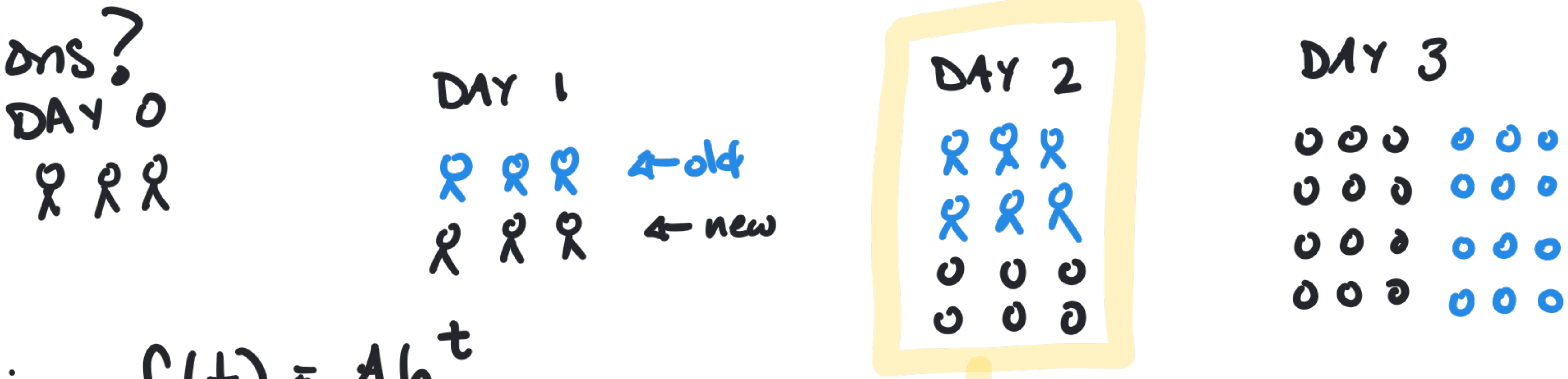
Shape of exponential function?

$f(x) = 2^{x}$

x	-2	-1	0	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



Example: 3 people are infected. Every day, each infected people infects one new person. How can we model & answer questions?



Form is $f(t) = Ab^t$

Know $f(0) = 3$ and $f(2) = 12$

So $f(0) = Ab^0 = 3 \Rightarrow A = 3$ and $f(2) = A \cdot b^2 \Rightarrow$

$12 = 3b^2 \Rightarrow$

$4 = b^2 \Rightarrow \boxed{b = 2}$

$$\boxed{f(t) = 3 \cdot 2^t}$$

← t is measured in days.

So, $f(t) = 3 \cdot 2^t$.

- How many infections are there...

  after one week?

  $f(7) = 3 \cdot 2^7 = 3 \cdot 128 = 384$ infections

  after one month? (31 days)

  $f(31) = 3 \cdot 2^{31} = 6{,}442{,}450{,}944$

  (World population (2018): 7,665,957,369)
  (Wikipedia)

  This is not a reasonable number of significant digits, either!

- When would we have 1000 infections?

  Need $f(t) = 1000 \Rightarrow 3 \cdot 2^t = 1000$

  $\Rightarrow 2^t = \frac{1000}{3}$

  $\Rightarrow \log_2(2^t) = \log_2\left(\frac{1000}{3}\right) \Rightarrow t = \log_2\left(\frac{1000}{3}\right)$

  ≈ 8.4 days.