Intro Video: Section 1.2
Essential Functions and a quick trigonometry review

Math F251X Fall 2020

Linear functions and linear models

Example:

$\frac{\Delta y}{\Delta x}$ is constant (the slope)

$32°F = 0°C$

$212°F = 100°C$

Let $C(T)$ be a function that takes as input temperature T, measured in °F, and returns $C(T)$, measured in °C

Slope? $\frac{\Delta y}{\Delta x} = \frac{100-0}{212-32} = \frac{100}{180} = \frac{10}{18} = \frac{5}{9}$

$C(T) = \frac{5}{9}(T-32)+0$ ← point-slope form using point $(32, 0)$.

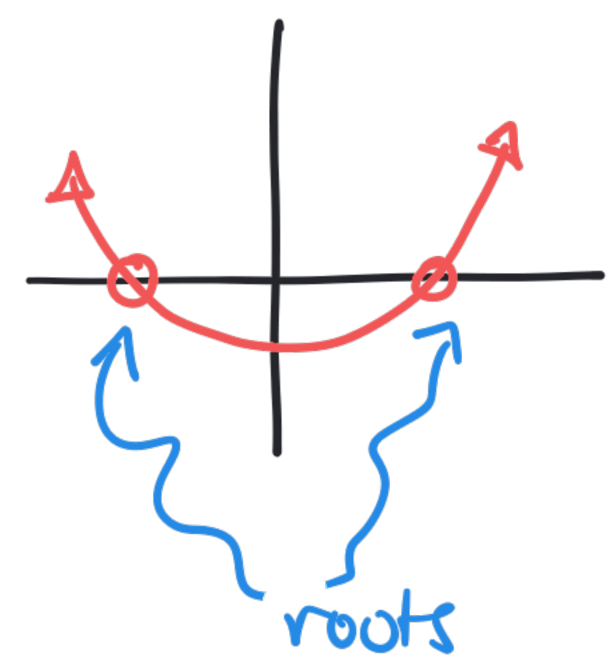
Polynomials

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

quadratic

$$f(x) = ax^2 + bx + c$$

__Roots__: where the function intersects the x-axis



Solve $f(x) = 0$ to find the root

Example

$$g(x) = (2x+3)(x-1)$$
$$= 2x^2 + x - 3$$

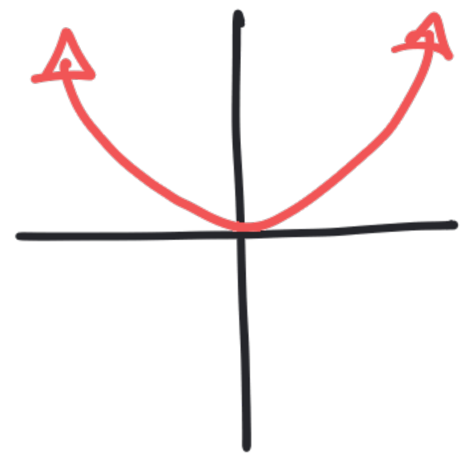
To find roots: solve $g(x) = 0$

$\Rightarrow (2x+3)(x-1) = 0 \Rightarrow 2x+3=0$ or $x-1=0$

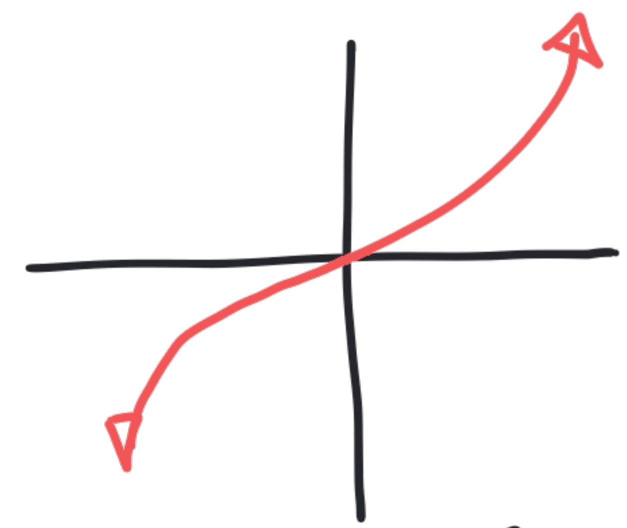
$\Rightarrow x = -3/2$ or $x = 1$

Or use the quadratic equation!

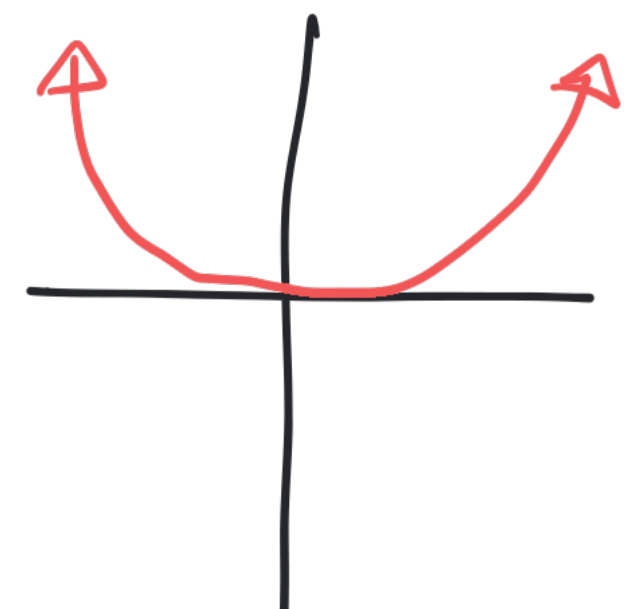
Functions whose graphs you should know



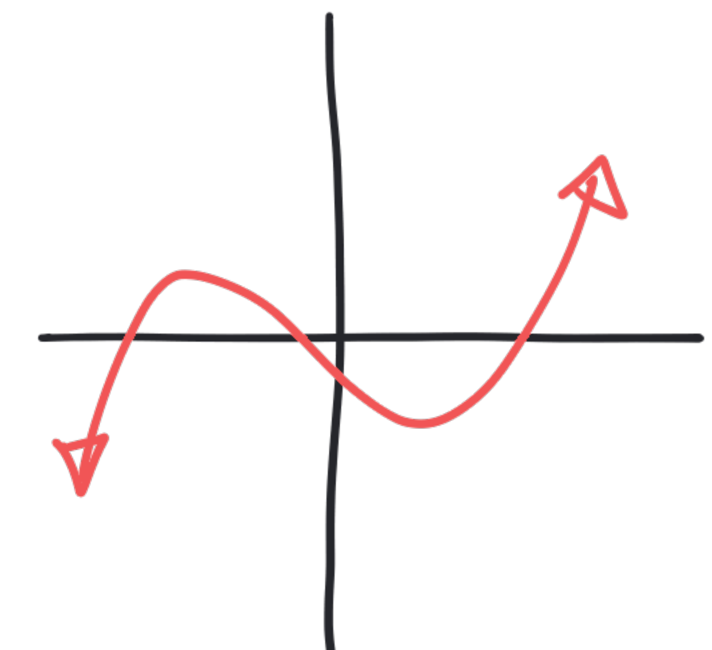
$f(x) = x^2$



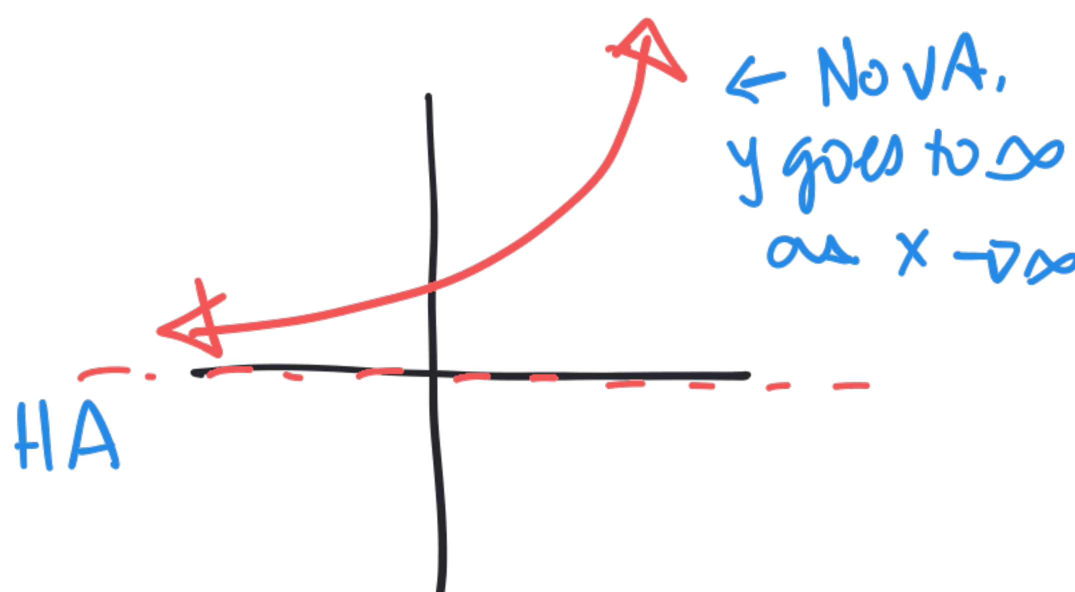
$f(x) = x^3$



$f(x) = x^4$

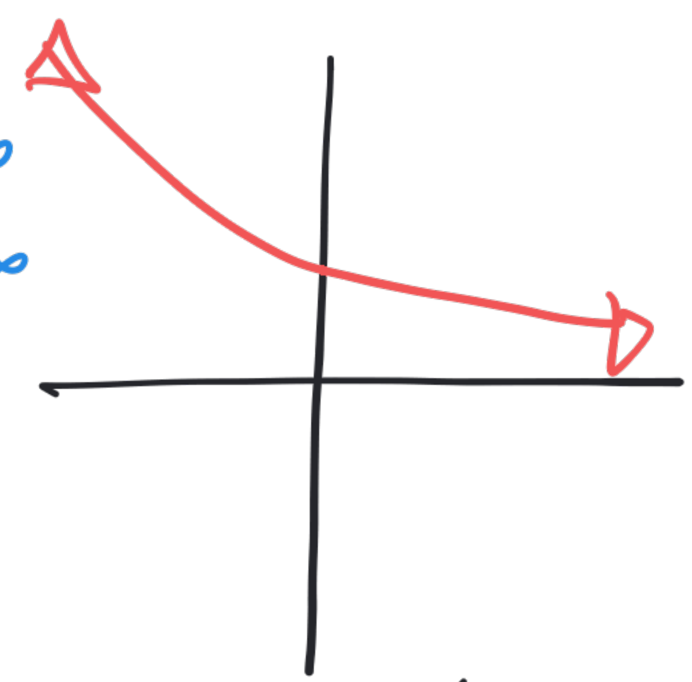


$f(x)$ = generic cubic



$f(x) = a^x$, $\boxed{a > 1}$

note $f(x) = e^x$ is in this collection



$f(x) = a^x$, $0 < a < 1$

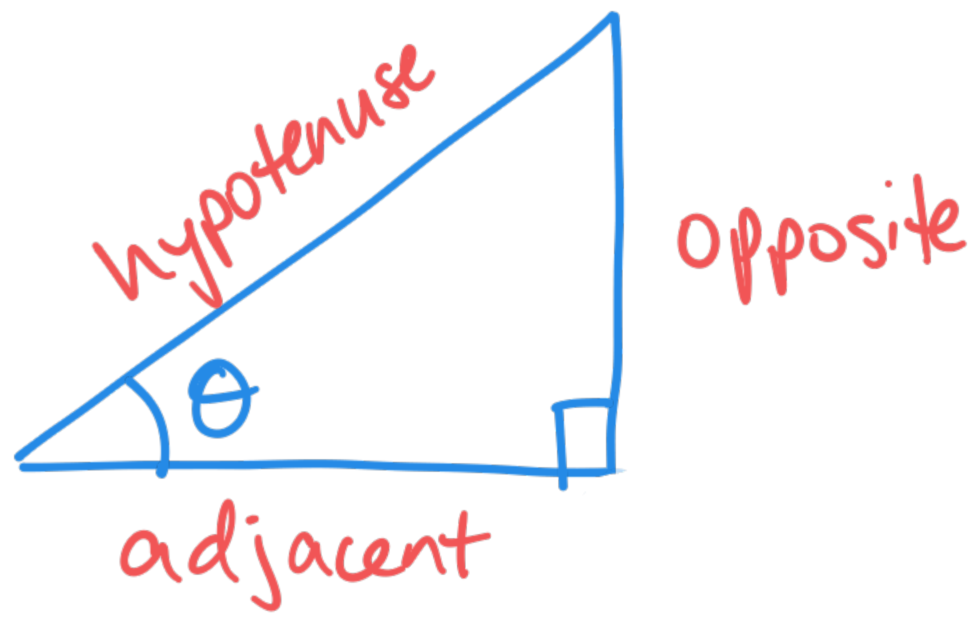
$f(x) = \ln(x)$

Quick Trigonometry Review

Three views of trigonometry:

1. Right Triangle Trigonometry
   → ratios of sides of triangles
2. Unit Circle Trigonometry
   → coordinates of points on the unit circle
3. As trigonometric <u>functions</u>

Right Triangle Trigonometry:

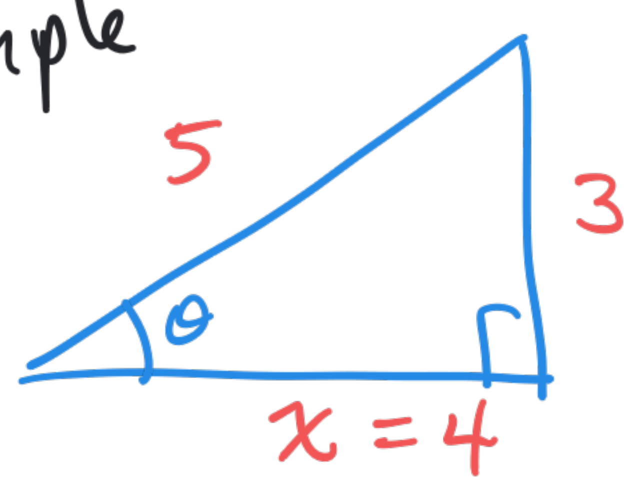


$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc(\theta) = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{\text{hyp}}{\text{adj}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot(\theta) = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}}$$

Example



What is x? Pythagorean Theorem:

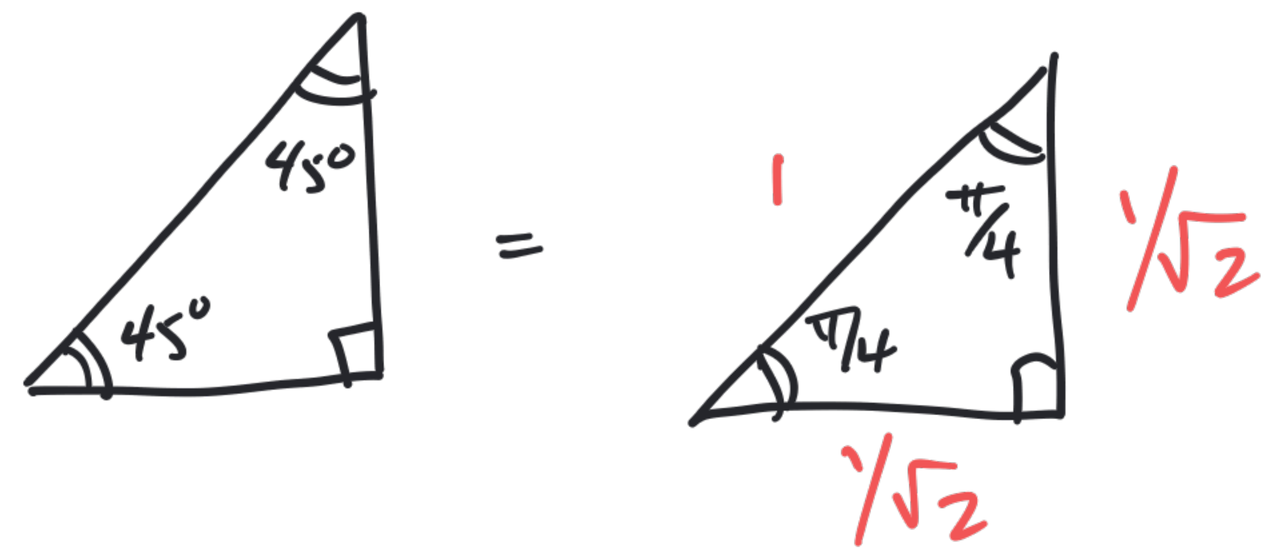
$3^2 + x^2 = 5^2 \Rightarrow 25 - 9 = x^2 \Rightarrow x^2 = 16$

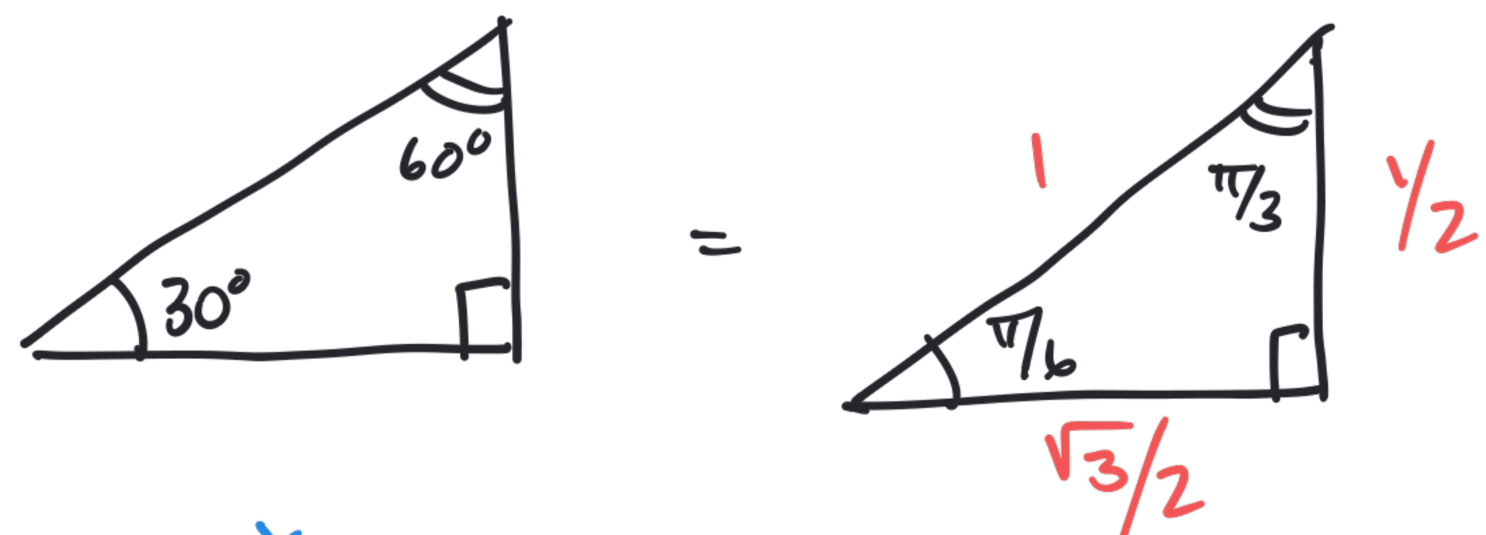
$x = 4$

What is $\cos(x)$? $\frac{4}{5}$

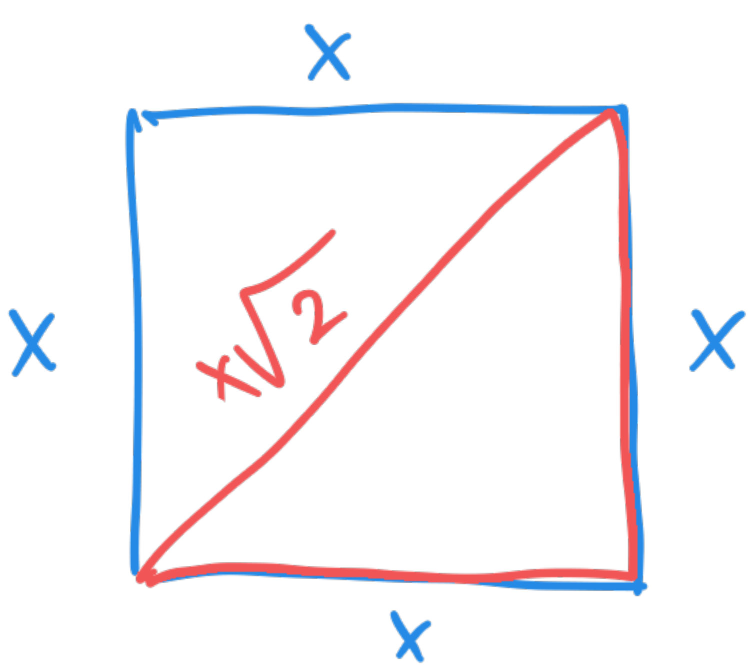
What is $\cot(x)$?

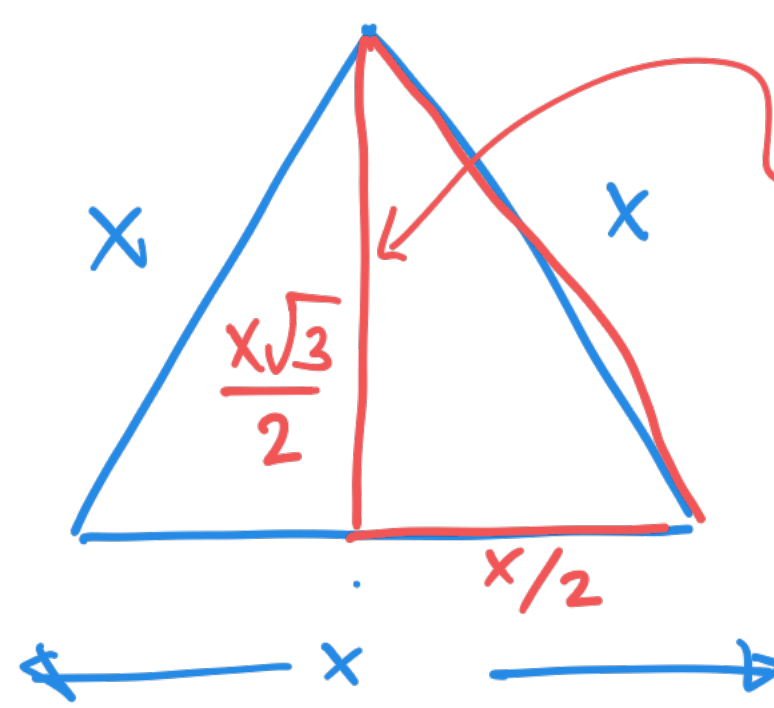
Memorize





derive side lengths from pythagorean theorem and geometry



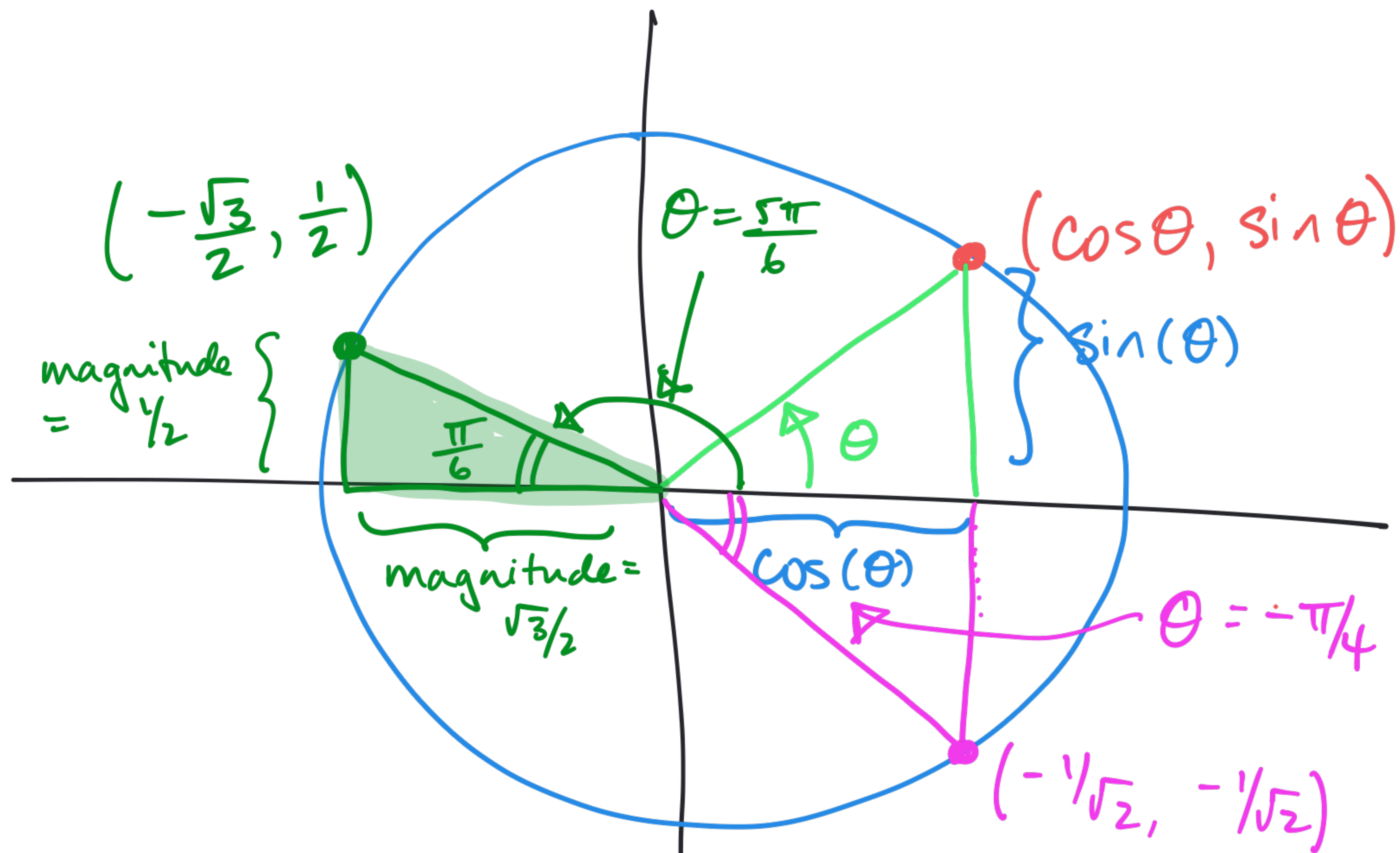


Solve $\left(\frac{x}{2}\right)^2 + ?^2 = x^2$

$$\Rightarrow \frac{x^2}{4} + ?^2 = x^2$$

$$\Rightarrow \frac{3x^2}{4} = ?^2 \Rightarrow ? = \frac{x\sqrt{3}}{2}$$

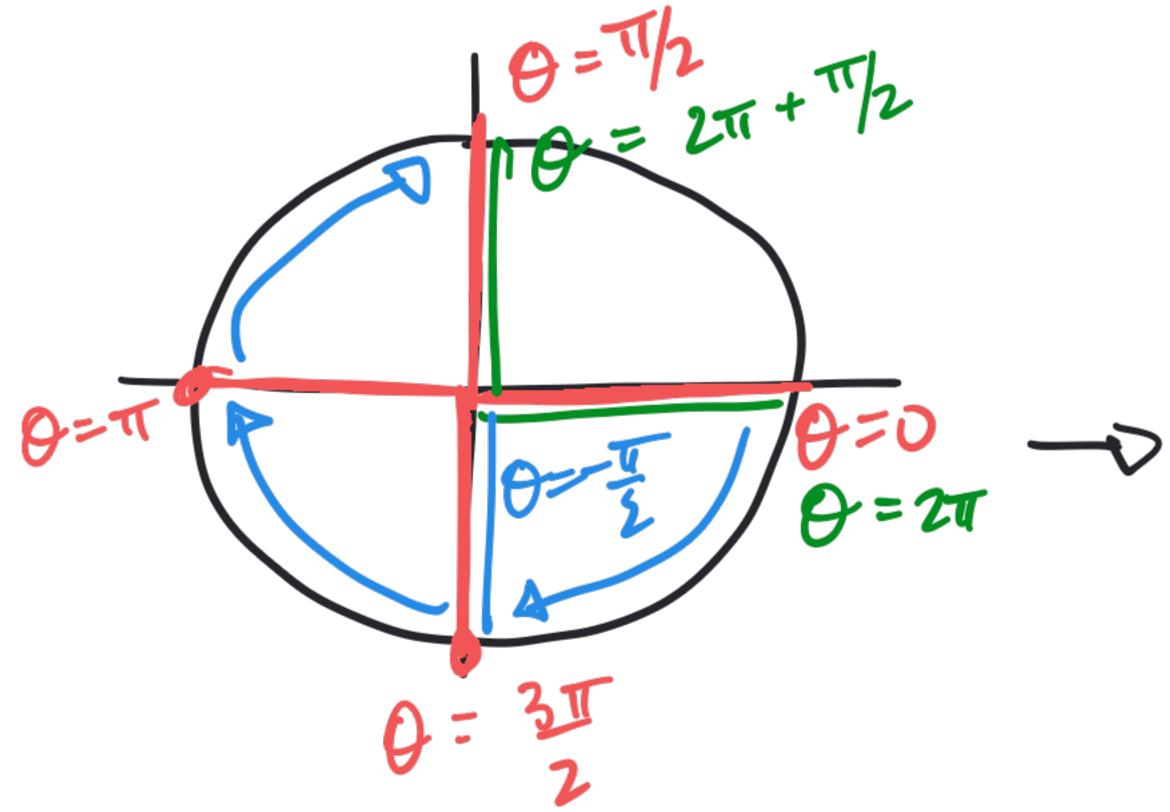
Unit Circle Trigonometry



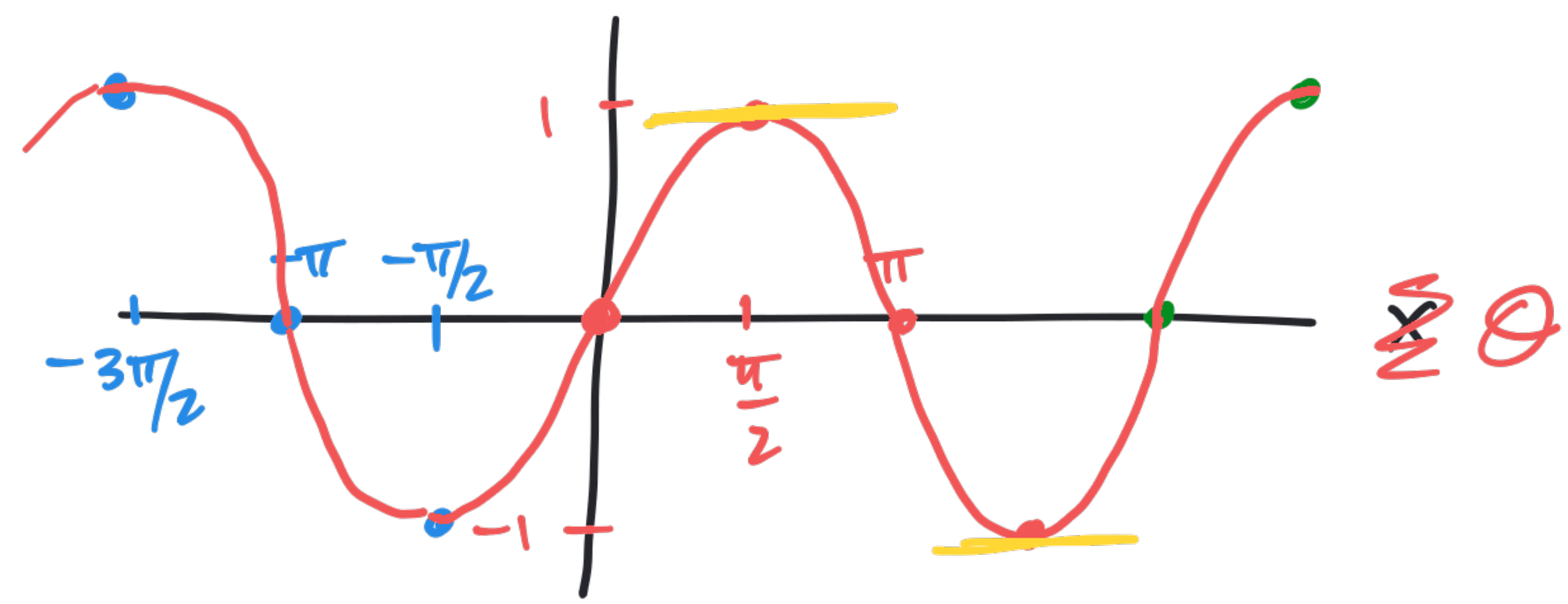
So $\cos\left(-\frac{\pi}{4}\right) = -1/\sqrt{2}$

$\sin\left(-\pi/4\right) = -1/\sqrt{2}$

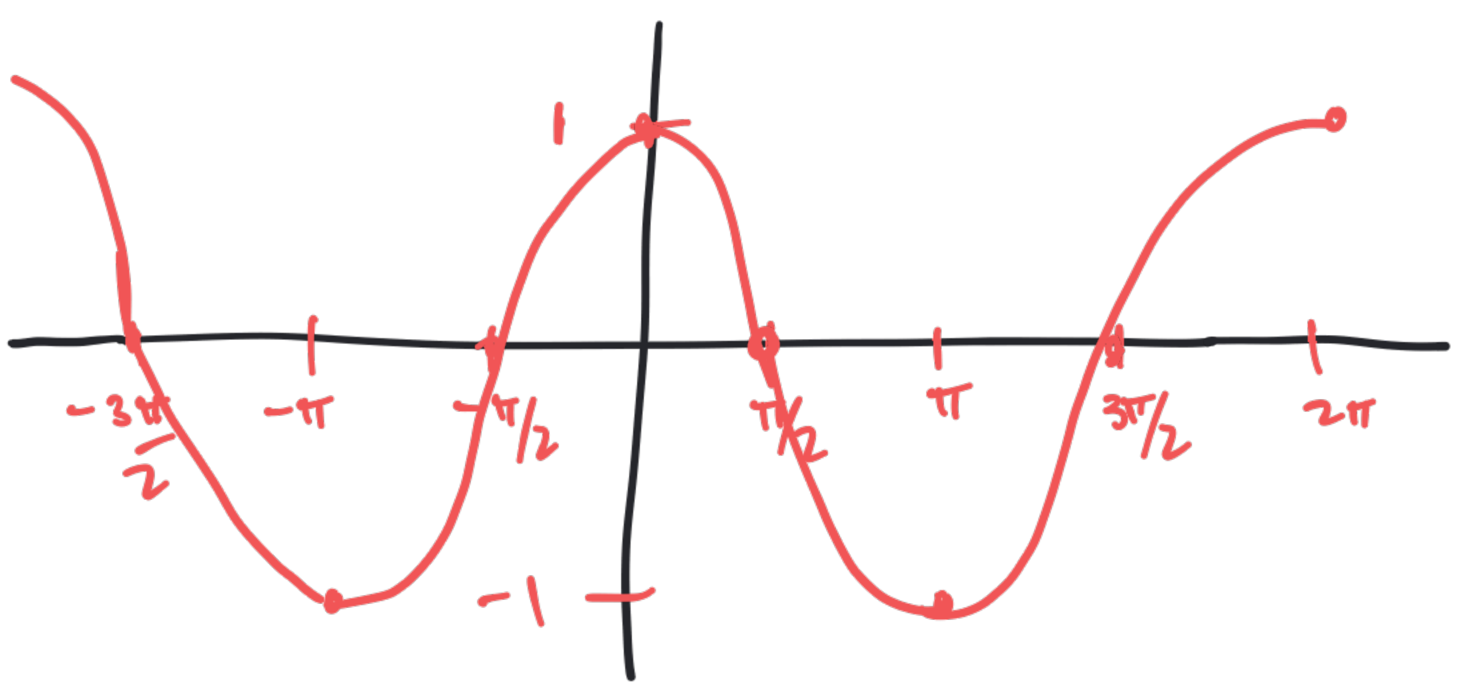
Unit Circle → trigonometric functions



sin(θ)



cos θ



$$\tan\theta = \frac{\sin(\theta)}{\cos(\theta)}$$