Final Review - Chapter 3
(Derivative Rules)

- Find derivatives using the limit defintion.
- Know how to apply the sum, difference, product, quotient, and chain rules.
- Know when to use logarithmic differentiation to find a derivative.

Example 1: Find the derivative of $f(x) = 9 + x - 2x^2$ using the definition of the derivative. Then find an equation of the tangent line at the point $(2, 3)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{x \to 0} \frac{(9 + (x+h) - 2(x+h)^2) - (9 + x - 2x^2)}{h}$$

$$= \lim_{h \to 0} \frac{9 + x + h - 2x^2 - 4xh - h^2 - 9 - x + 2x^2}{h} = \lim_{h \to 0} \frac{h - 4xh - h^2}{h} = \lim_{h \to 0} 1 - 4x - h$$

$$= 1 - 4x.$$

So $f'(2) = 1 - 4 \cdot 2 = 1 - 8 = -7 = m$

equation: $y - 3 = -7(x - 2)$

Example 2: Calculate y'.

a) $y = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt[5]{x^3}} = x^{-1/2} - x^{-3/5}$

$$y' = -\frac{1}{2} x^{-3/2} + \frac{3}{5} x^{-8/5}$$

b) $y = \frac{\tan x}{1 + \cos x}$

quotient rule:

$$y' = \frac{(1 + \cos x)(\sec^2 x) - (\tan x)(-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{(1 + \cos x)(\sec^2 x) + \sin x \tan x}{(1 + \cos x)^2}$$

Example 3: Calculate y'.

a) $y = x\cos^{-1}x$

$$y' = 1\cdot\cos^{-1}x + x\cdot\frac{-1}{\sqrt{1-x^2}}$$

$$= \arccos x - \frac{x}{\sqrt{1-x^2}}$$

b) $y = (\arcsin(2x))^2$

$$y' = 2(\arcsin(2x))^1\cdot\frac{1}{\sqrt{1-(2x)^2}}\cdot 2$$

$$= \frac{4\arcsin(2x)}{\sqrt{1-4x^2}}$$

Example 4: Calculate y'.

a) $y = e^{x\sec x}$

$$y' = \left(e^{x\sec x}\right)(1\cdot\sec x + x\cdot\sec x\tan x)$$

$$= (\sec x)(1+x\tan x)e^{x\sec x}.$$

b) $y = 10^{\tan(\pi\theta)}$

$$y' = \ln 10\cdot 10^{\tan(\pi\theta)}\cdot\sec^2(\pi\theta)\cdot\pi$$

$$y' = \pi\ln 10\sec^2(\pi\theta)\cdot 10^{\tan(\pi\theta)}$$

Example 6: Find $\frac{dy}{dx}$.

a) $y = \arcsin(e^{2x})$

$$y' = \frac{1}{\sqrt{1-(e^{2x})^2}}\cdot e^{2x}\cdot 2$$

$$= \frac{2e^{2x}}{\sqrt{1-e^{4x}}}$$

b) $y = \int_{x^2}^{3}\frac{t+4}{\cos t}dt = -\int_{3}^{x^2}\frac{t+4}{\cos t}dt$

$$\frac{dy}{dx} = -\left(\frac{x^2+4}{\cos(x^2)}\right)\cdot 2x = \frac{-2x(x^2+4)}{\cos(x^2)}$$

- Find derivatives using implicit differentiation.

Example 5: Given $xe^y = y\sin x$ find y'.

$$1\cdot e^y + x\cdot e^y\cdot y' = y'\cdot \sin x + y\cos x$$

$$xe^y y' - (\sin x)y' = y\cos x - e^y$$

$$y' = \frac{y\cos x - e^y}{xe^y - \sin x}$$

Example 6: Given $y - x\cos y = x^2 y$ find y'

$$y' - 1\cdot\cos y - x\cdot(-\sin y)\cdot y' = 2xy + x^2 y'$$

$$y' + (x\sin y)y' - x^2 y' = 2xy + \cos y$$

$$(1 + x\sin y - x^2)y' = 2xy + \cos y$$

$$y' = \frac{2xy + \cos y}{1 + x\sin y - x^2}$$

$x^2 - 4 = (x-2)(x+2)$

Example 7: Find the derivative of $h(x) = \ln\left(\frac{x^2-4}{2x+5}\right) = \ln(x^2-4) - \ln(2x+5)$

$$= \ln(x-2) + \ln(x+2) - \ln(2x+5)$$

$$h'(x) = \frac{1}{x-2} + \frac{1}{x+2} - \frac{2}{2x+5}$$

Example 8: Find the derivative of $y = (\cos x)^x$

(take ln both sides)

$\ln y = x \ln \cos x$

$\frac{1}{y} y' = 1 \cdot \ln(\cos x) + x \frac{1}{\cos x} \cdot -\sin x$

$y' = y\left[\ln(\cos x) - \frac{\sin x}{\cos x}\right] = \left((\cos x)^x\right)\left[\ln(\cos x) - x \tan x\right]$

Example 9: Find the derivative of $y = (x+4)^{\tan(2x)}$

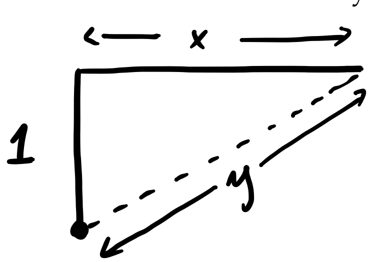
$\ln y = (\tan 2x) \ln(x+4)$

$\frac{1}{y} \cdot y' = (\sec^2 2x)(2) \cdot \ln(x+4) + \tan(2x) \cdot \frac{1}{x+4}$

$y' = \Big(\underbrace{(x+4)^{\tan(2x)}}_{y}\Big)\left(2\sec^2(2x)\ln(x+4) + \frac{\tan(2x)}{x+4}\right)$

- Solve related rates problems.

Example 10: A plane flying horizontally at an altidue of 1 mile and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.



Find $\frac{dy}{dt}$ when $y = 2$, $\frac{dx}{dt} = 500$

$y^2 = 1^2 + x^2$

$2y \frac{dy}{dt} = 2x \frac{dx}{dt}$

$\frac{dy}{dt} = \boxed{\frac{x}{y}} \cdot \frac{dx}{dt}$

we need x when y = 2 :

So $2^2 = 1^2 + x^2$

So $x = \sqrt{3}$

So,

$\frac{dy}{dt} = \frac{\sqrt{3}}{2} 500 = 250\sqrt{3}$ mi/h

Example 11: The sides of an equilaterial triangle are increasing at a rate of 10 cm/min. At what rate is the area of the triangle increasing when the sides are 30 cm long? ($A = \frac{\sqrt{3}}{4}(\text{side})^2$)

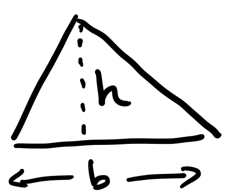
← Oooh! That makes it easy!

$$A = \frac{\sqrt{3}}{4} s^2$$

We want $\frac{dA}{dt}$ when $s = 30$ cm assuming $\frac{ds}{dt} = 10$ cm/min

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2} s \cdot \frac{ds}{dt} = \frac{\sqrt{3}}{2} \cdot 30 \cdot 10 = 150\sqrt{3}\ \text{cm}^2/\text{min}$$

Example 12: The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?



$A = \frac{1}{2} bh$

Given: $\frac{dh}{dt} = 1$ $\frac{dA}{dt} = 2$

Find $\frac{db}{dt}$ when $A = 100$ and $h = 10$.

Find b: $100 = \frac{1}{2} \cdot 10 \cdot b$

So $b = 20$.

$$\frac{dA}{dt} = \frac{1}{2}\left(\boxed{b} \cdot \frac{dh}{dt} + \frac{db}{dt} \cdot h\right)$$

Plug in:

$$2 = \frac{1}{2}\left(20 \cdot 1 + \frac{db}{dt} \cdot 10\right) = 10 + 5\frac{db}{dt}$$

So $\frac{db}{dt} = \frac{-8}{5}$ cm/min (The base is decreasing.)

Suggested Probs: Ch. 2 Rvw, Pg 165 # 39 (a & b) and Ch. 3 Rvw, Pg 267 # 1-39 every other odd, 57, 97, 99