RECITATION 10: 4-2 HOW DERIVATIVES AFFECT THE SHAPE OF A GRAPH (PART 2)

WARM-UP QUESTIONS:

1. Given a function $y = f(x)$ how do you...
   (a) determine where f is increasing/decreasing?

   (b) use f' to identify any local maximum and minimum values?

   (c) determine where f is concave up or concavedown?

   (d) find inflection points?

2. Let $f(x) = x - 2\cos x$ be restricted to the interval $I = [0, 4\pi]$.
   (a) Determine intervals of increase and decrease of f on I.

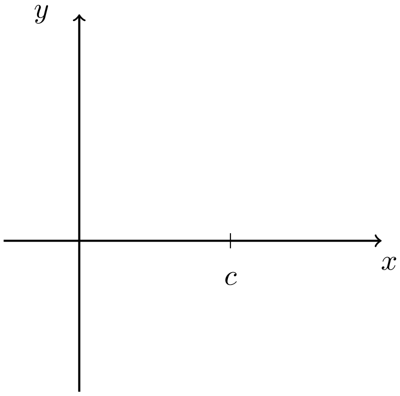
   (b) Use the First Derivative Test to identify any local maximums or minimums of f on I.

   (c) Graph $f(x)$ on your calculator to check your answer is correct.

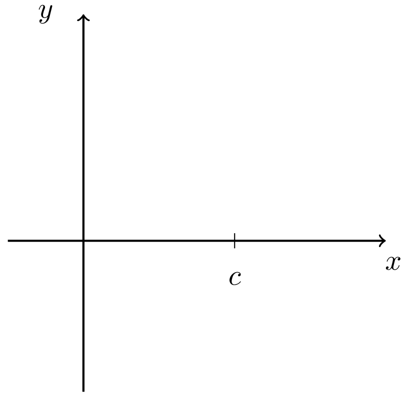
$\boxed{\text{MOTIVATING EXAMPLES:}}$

Assume $f(x)$ is differentiable (and therefore continuous) for all real numbers. On the axes below, sketch a graph of $f(x)$ with the given property.

(i) $f(x)$ has a local maximum at $x = c$



(ii) $f(x)$ has a local minimum at $x = c$



$\boxed{\text{QUESTION 1:}}$ What can you say about $f'(c)$ in picture (i)? picture (ii)? Is it the same for your neighbors pictures? $\boxed{\text{QUESTION 2:}}$ What can you say about $f''(c)$ in picture (i)? picture (ii)? Is it the same for your

neighbors pictures?

> THE SECOND DERIVATIVE TEST: Suppose f'' is continuous near c.
>
> a) If $f'(c)$_______ and $f''(c)$_______, then f has a local minimum at c.
>
> b) If $f'(c)$_______ and $f''(c)$_______, then f has a local maximum at c.

$\boxed{\text{QUESTION 3:}}$ What happens if $f'(c) = 0$ and $f''(c) = 0$? Can you draw any conclusions about whether f has a local max or min? Why"

Example 2: Find the local maximum and minimum values of the functions. Choose either the first or second derivative test. Explain why you made the choice that you did.

(a) $f(x) = x^4 - 4x + 3$

(b) $f(x) = \dfrac{x}{x^2 + 2}$

Example 3: Sketch a possible graph of a function f that satisfies the following conditions:

(i) $f(0) = 0$, $f(2) = 3$, $f(4) = 6$, $f'(0) = f'(4) = 0$.

(ii) $f'(x) > 0$ for $0 < x < 4$ and $f'(x) < 0$ for $x < 0$ and for $x > 4$.

(iii) $f''(x) > 0$ for $x < 2$ and $f''(x) < 0$ for $x > 2$.

Example 4: Given the function $f(x) = \ln(x^2 + 4)$ find the following.

(a) Find the intervals of increase or decrease.

(b) Find the local maximum and minimum values.

(c) Find the intervals of concavity and inflection points.

(d) Use the information to sketch the graph.

Example 5: Given the function $f(x) = 5x^{2/3} - 2x^{5/3}$, find the following.

(a) Find the intervals of increase or decrease.

(b) Find the local maximum and minimum values.

(c) Find the intervals of concavity and inflection points.

(d) Use the information to sketch the graph.

Example 6: Suppose the function $f(t) = t^3 - 12t + 2$ describes the motion of a particle along the x-axis for $t \geq 0$. Find $f'(2)$ and $f''(2)$. Is the velocity of the particle increase or decreasing at $t = 2$? Explain your answer in complete sentences.

Example 7: An economist announces that the national deficit is increasing, but at a decreasing rate. Interpret this statement in terms of a function and its first and second derivatives.

Example 8: Let $f(t)$ be the temperature at time t where you live and suppose at time $t = 3$ you feel uncomfortably cold. How do you feel about the given data in each case?

a) $f'(3) = 2$, $f''(3) = -4$

b) $f'(3) = -2$, $f''(3) = -4$