RECITATION 12: REVIEW OF CHAPTERS 3 & 4

For this worksheet, you want to focus on more than merely "getting the right answer" especially since you have no reason to expect these particular problems will appear on the midterm. What else should you do while working through these problems?

1. Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 1$ where the tangent is horizontal.

2. A particle moves on a vertical line so that it's coordinate at time t is $y = t^3 - 12t + 3$ for $t \geq 0$. Assume t is in seconds and s is in meters.

   (a) Find the velocity and the acceleration of the particle at time t.

   (b) When will the velocity be zero?

   (c) Sketch a diagram of the motion of the particle.

3. The angle of elevation of the sun is decreasing at a rate of 0.25 radian per hour. How fast is the shadow cast by a 400-ft-tall building increasing when the angle of elevation of the sun is $\pi/6$?

4. The radius of a circular disc is given as 24 cm with a maximum error in measurement of 0.2 cm.

   (a) Use differentials to estimate the maximum error in the calculated area of the disk.

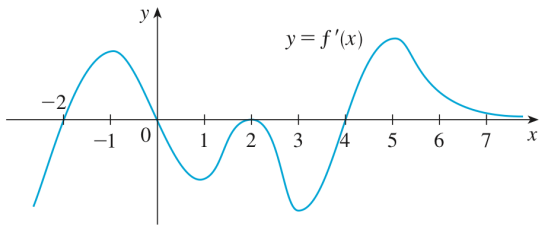
   (b) What is the *relative* error?

5. Find the critical numbers of $f(t) = \frac{\sqrt{t}}{1+t^2}$ in $[0, 2]$.

6. (a) Explain how Rolle's Theorem is a special case of the Mean Value Theorem.

(b) Verify that the function $f(x) = \dfrac{x}{x+2}$ satisfies the two hypotheses of the Mean Value Theorem on the interval $[1, 4]$. Then, find all numbers c that satisfy the conclusion of the Mean Value Theorem.

7. The graph of the first derivative f' of a function f is shown below.



(a) On what intervals is f increasing?

(b) At what values of x does f have a local maximum or minimum?

(c) On what intervals is f concave up or concave down?

(d) What are the x-coordinates of the inflection points of f?

8. Evaluate the following limits. Show your work.

(a) $\lim_{x\to\infty} \frac{(\ln x)^2}{e^{2x}}$

(b) $\lim_{x\to 0^+} x \cdot \ln x$

(c) $\lim_{x\to 0^+} x^{\ln x}$

(d) $\lim_{x\to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

9. Use the Second Derivative Test to identify any local maximums or minimums of $f(x) = 2x(20-x^2)$.

10. Sketch the graph of $f(x) = \dfrac{1}{x^2 - 9}$ incorporating the information below.

(a) Find the domain.

(b) Find the x and y-intercepts.

(c) Find the symmetries/ periodicity of the curve.

(d) Determine the asymptotes.

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

(g) Find the intervals of concavity/inflection points.

(h) Sketch the curve.

11. A cylindrical can is to be constructed with a volume of 20 cubic feet. The material for the top of the can costs \$2 per square foot. The material for the bottom of the can costs \$10 per square foot. The material used to construct the side of the can costs \$ 5 per square foot. What dimensions of the can will minimize the costs of materials?