Recitation: 5-3 The Fundamental Theorem of Calculus (Part 2)

Review: Look back at your notes from class yesterday and state both parts of the Fundamental Theorem of Calculus below.

- **The Fundamental Theorem of Calculus, Part 1**

- **The Fundamental Theorem of Calculus, Part 2**

Example 1: Using the FTC (Part ___), differentiate the following functions.

(a) $f(x) = \int_0^x \sqrt{1+t^2}dt$

(b) $g(x) = \int_{\cos x}^1 (t + \sin t)dt$

Example 2: Using the FTC (Part ___), differentiate $h(x) = \int_{\cos x}^{5x} \cos(u^2)du$

Example 3: Use the second part of the FTC to evaluate the integral, or explain why it does not exist.

(a) $\int_0^4 y(3+2y-y^2)dy$

(b) $\int_1^8 \sqrt[3]{x}dx$

Example 4: Use the second part of the FTC to evaluate the integral, or explain why it does not exist.

(a) $\int_{-1}^7 x^{-3}dx$

(b) $\int_{\pi/4}^{5\pi/2} \sin x dx$

Example 5: Use the second part of the FTC to evaluate the integral, or explain why it does not exist.

(a) $\int_1^3 (x-2)(x+3)dx$

(b) $\int_1^2 \left(x+\frac{1}{x}\right)^2 dx$

Example 6: Use the second part of the FTC to evaluate the integral, or explain why it does not exist.

(a) $\displaystyle\int_0^{\pi/4} \sec^2 x dx$

(b) $\displaystyle\int_1^8 \frac{x^2-1}{x} dx$

Example 7: Use the second part of the FTC to evaluate the integral, or explain why it does not exist.

(a) $\displaystyle\int_1^4 \sqrt{2} dx$

(b) $\displaystyle\int_1^9 \sqrt{\frac{2}{x}} dx$

Example 8: If $f(1) = 10$, f' is continuous, and $\displaystyle\int_1^5 f'(x)dx = 23$, what is the value of $f(5)$?

Example 9: Determine whether the statement is true or false. If either case, explain why or give an example that disproves the statement.

(a) $\displaystyle\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

(b) $\displaystyle\int_a^b [f(x)g(x)]dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$

Example 10: Determine whether the statement is true or false. If it is true, explain why or give an example that disproves the statement.

(a) $\displaystyle\int_a^b \sqrt{f(x)}dx = \sqrt{\int_a^b f(x)dx}$

(b) $\displaystyle\int_{-2}^1 \frac{1}{x^4} = -\frac{3}{8}$

Example 11: Evaluate the limits by first recognizing the sum as a Reimann sum for a function.

(a) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5 + \frac{2i}{n}\right)^{10}$

(b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n} \sin\left(2 + \frac{5i}{n}\right)$