Recitation: 5-4 Indefinite Integrals and the Net Change Theorem (Part 2)

Fill in the following anti-derivatives.

All the indefinite integrals you (should) already know:

- $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sec^2 x dx = \tan x + C$
- $\int \csc^2 x dx = -\cot x + C$
- $\int \sec x \tan x = \sec x + C$
- $\int \csc x \cot x = -\csc x + C$
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{1}{\ln a} a^x + C$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
- $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

- **Question 1:** How do you check your answers when computing integrals? For example, suppose $\int f(x)dx = F(x) + C$. How do you know you are right?

  Find $\frac{d}{dx}(F(x)+C)$. This should give you $f(x)$.

- **Question 2:** For what value of n does the reverse power rule for the antiderivative of x^n not apply, and what is the antiderivate of x^n for this value of n?

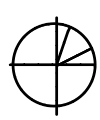
  for $n=-1$. In this case $\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$

- **Question 3:** You have no product or quotient rule for anti-derivatives. How then do you deal with a function that is a product or a quotient?

  You simplify by multiplying or dividing and manipulate the integrand (stuff inside $\int$ sign) until you can use rules you know.

Example 1: Evaluate the following integrals.

(a) $\int_{1/2}^{\sqrt{3}/2} \frac{6}{\sqrt{1-t^2}} dt$



$= 6 \cdot \sin^{-1}(t) \Big|_{1/2}^{\sqrt{3}/2}$

$= 6\sin^{-1}(\sqrt{3}/2) - 6\sin^{-1}(1/2)$

$= 6(\pi/3) - 6(\pi/6)$

$= 2\pi - \pi$

$= \boxed{\pi}$

(b) $\int_1^2 \frac{4+u^2}{u^3} du = \int_1^2 \left(\frac{4}{u^3} + \frac{u^2}{u^3}\right) du$

$= \int_1^2 (4u^{-3} + 1/u)\, du$

$= \left(\frac{4u^{-2}}{-2} + \ln|u|\right)\Big|_1^2$

$= \left(-2/u^2 + \ln|u|\right)\Big|_1^2$

$= (-2/4 + \ln 2) - (-2 + \ln 1)$

$= \boxed{\frac{3}{2} + \ln 2}$

Example 2: Evalute the following integrals.

(a) $\int_0^1 (3 + x\sqrt[3]{x})dx = \int_0^1 (3 + x \cdot x^{1/3})dx$

$= \int_0^1 (3 + x^{4/3})dx$

$= \left(3x + \frac{3}{7}x^{7/3}\right)\Big|_0^1$

$= 3 + 3/7$

$= 21/7 + 3/7$

$= \boxed{24/7}$

(b) $\int_0^{\pi/4} \sec x(\sec x + \tan x)dx = \int_0^{\pi/4} (\sec^2 x + \sec x \tan x)\,dx$

$= (\tan x + \sec x)\Big|_0^{\pi/4}$

$= \tan \pi/4 + \sec \pi/4 - (\tan 0 + \sec 0)$

$= \frac{\sin \pi/4}{\cos \pi/4} + \frac{1}{\cos \pi/4} - \left(\frac{\sin 0}{\cos 0} + \frac{1}{\cos 0}\right)$

$= \frac{\sqrt{2}/2}{\sqrt{2}/2} + \frac{1}{(\sqrt{2}/2)} - (0/1 + 1/1)$

$= 1 + 2/\sqrt{2} \; \frac{\sqrt{2}}{\sqrt{2}} - 1$

$= \boxed{\sqrt{2}}$

Example 3: Find the general indefinite integral.

(a) $\int v(3v-2)^2 dv = \int v(9v^2 - 12v + 4)\,dv$

$= \int (9v^3 - 12v^2 + 4v)dv$

$= \frac{9v^4}{4} - \frac{12}{3}v^3 + \frac{4}{2}v^2 + c$

$= \boxed{\frac{9}{4}v^4 - 4v^3 + 2v^2 + c}$

(b) $\int (\csc^2 t - 2e^t)dt = \boxed{-\cot t - 2e^t + c}$

Example 4: Find the general indefinite integral.

(a) $\int (\sqrt{x^3} + \sqrt[3]{x^2})dx = \int \left(x^{3/2} + x^{2/3}\right)dx$

$= \boxed{\frac{2}{5}x^{5/2} + \frac{3}{5}x^{5/3} + c}$

(b) $\int \sqrt{\frac{5}{x}}\,dx = \int \sqrt{5}\, x^{-1/2} dx$

$= \sqrt{5}\; 2x^{1/2} + c$

$= 2\sqrt{5}\sqrt{x} + c$

$= \boxed{2\sqrt{5x} + c}$

Example 5: Find the general indefinite integral.

a) $\int \left(\frac{1+2x}{x}\right)^2 dx = \int \frac{1+4x+4x^2}{x^2}\,dx$

$= \int \left(\frac{1}{x^2} + \frac{4x}{x^2} + \frac{4x^2}{x^2}\right) dx$

$= \int \left(x^{-2} + \frac{4}{x} + 4\right) dx$

$= -x^{-1} + 4\ln|x| + 4x + C$

$= \boxed{-\frac{1}{x} + 4\ln|x| + 4x + C}$

b) $\int (2 + \tan^2\theta)\,d\theta = \int (2 + \sec^2\theta - 1)\,d\theta$

change into $\sec^2\theta$ using trig

?

$\sin^2\theta + \cos^2\theta = 1$

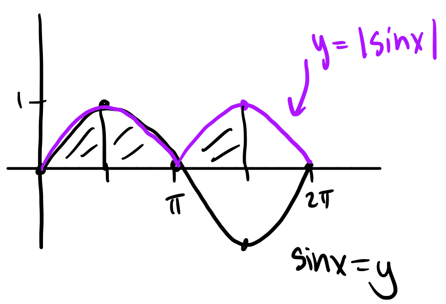
$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$= \int (1 + \sec^2\theta)\,d\theta$

$= \boxed{\theta + \tan\theta + C}$

Example 6: Sketch a picture of $\int_0^{3\pi/2} |\sin x|dx$. Use symmetry to find the area.



$\int_0^{3\pi/2} |\sin x|\,dx = 3\int_0^{\pi/2} \sin x\,dx$

$= (-3\cos x)\Big|_0^{\pi/2}$

$= -3\cos(\pi/2) + 3\cos 0$

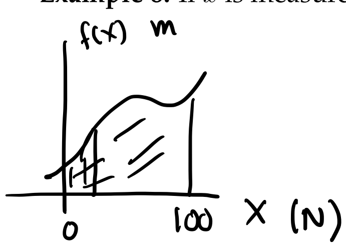
$= -3(0) + 3(1)$

$= \boxed{3}$

Example 7: If oil leaks from a tank at a rate of $r(t)$ gallons per minute, what does $\int_0^{120} r(t)$ dt represent? Give your answer with proper units.

This tells us the amount of oil that has leaked from the tank (in gallons) during the first 120 minutes.

Example 8: If x is measured in meters and $f(x)$ is measured in newtons, what are the units for $\int_0^{100} f(x)dx$?



units are Newton * meter

(also known as a Joule)

- Question 4: How is displacement different from distance traveled?

Displacement is position @ start − position @ end. It tells you how the particle has moved relative to its starting position.

Distance tells you how far the particle traveled.

- Question 5: Given a velocity function $v(t)$ on an interval $[a, b]$, how do you compute displacement? How do you compute distance traveled?

Displacement $= \int_a^b v(t)\,dt$

Distance $= \int_a^b |v(t)|\,dt$

this will probably need to be broken into many parts.

Example 9: Suppose $v(t) = t^2 - 4$ for $0 \le t \le 3$ describes a particle moving along a line. Find the

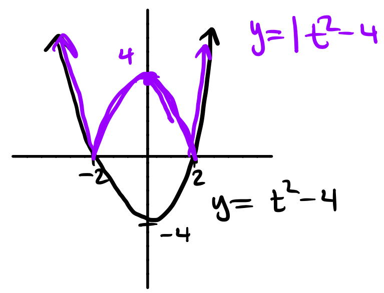
(a) displacement of the particle.

$$\text{displacement} = \int_0^3 (t^2 - 4)\,dt$$
$$= \left(\tfrac{1}{3}t^3 - 4t\right)\Big|_0^3$$
$$= \tfrac{1}{3}(27) - 12$$
$$= 9 - 12$$
$$= \boxed{-3\text{ m}}$$

The particle is 3 m to the left of where it began.

(b) distance traveled by the particle.

graph!



$$\text{dist} = \int_0^3 |t^2 - 4|\,dt$$
$$= \int_0^2 -(t^2 - 4)\,dt + \int_2^3 (t^2 - 4)\,dt$$
$$= \int_0^2 (4 - t^2)\,dt + \int_2^3 (t^2 - 4)\,dt$$

that (−) flips the normal parabola up!

$$= \left(4t - \tfrac{1}{3}t^3\right)\Big|_0^2 + \left(\tfrac{1}{3}t^3 - 4t\right)\Big|_2^3$$
$$= \left(8 - 8/3\right) + \left(27/3 - 12\right) - \left(8/3 - 8\right)$$
$$= 16 - 16/3 - 3$$
$$= 13\left(\tfrac{3}{3}\right) - 16/3$$
$$= 39/3 - 16/3$$
$$= \boxed{23/3 = 7.\overline{6}\ \text{m}}$$