Recitation: 5-4 Indefinite Integrals and the Net Change Theorem (Part 2)

Fill in the following anti-derivatives.

All the indefinite integrals you (should) already know:

- $\int x^n dx =$
- $\int \sin x dx =$
- $\int \cos x dx =$
- $\int \sec^2 x dx =$
- $\int \csc^2 x dx =$
- $\int \sec x \tan x dx =$
- $\int \csc x \cot x dx =$
- $\int \frac{1}{x} dx$
- $\int e^x dx$
- $\int a^x dx$
- $\int \frac{1}{\sqrt{1-x^2}} dx$
- $\int \frac{1}{1+x^2} dx$

- **Question 1:** How do you check your answers when computing integrals? For example, suppose $\int f(x)dx = F(x) + C$. How do you know you are right?
- **Question 2:** For what value of n does the reverse power rule for the antiderivative of x^n not apply, and what is the antiderivate of x^n for this value of n?
- **Question 3:** You have no product or quotient rule for anti-derivatives. How then do you deal with a function that is a product or a quotient?

Example 1: Evaluate the following integrals.

(a) $\displaystyle\int_{1/2}^{\sqrt{3}/2} \frac{6}{\sqrt{1-t^2}} dt$

(b) $\displaystyle\int_1^2 \frac{4+u^2}{u^3} du$

Example 2: Evalute the following integrals.

(a) $\int_0^1 (3 + x\sqrt[3]{x})dx$

(b) $\int_0^{\pi/4} \sec x(\sec x + \tan x)dx$

Example 3: Find the general indefinite integral.

(a) $\int v(3v-2)^2 dv$

(b) $\int (\csc^2 t - 2e^t)dt$

Example 4: Find the general indefinite integral.

(a) $\int (\sqrt{x^3} + \sqrt[3]{x^2})dx$

(b) $\int \sqrt{\frac{5}{x}}dx$

Example 5: Find the general indefinite integral.

a) $\displaystyle\int \left(\frac{1+2x}{x}\right)^2 dx$

b) $\displaystyle\int (2+\tan^2\theta)d\theta$

Example 6: Sketch a picture of $\displaystyle\int_0^{3\pi/2} |\sin x|dx$. Use symmetry to find the area.

Example 7: If oil leaks from a tank at a rate of $r(t)$ gallons per minute, what does $\displaystyle\int_0^{120} r(t)$ dt represent? Give your answer with proper units.

Example 8: If x is measured in meters and $f(x)$ is measured in newtons, what are the units for $\int_0^{100} f(x)dx$?

- Question 4: How is displacement different from distance traveled?

- Question 5: Given a velocity function $v(t)$ on an interval $[a, b]$, how do you compute displacement? How do you compute distance traveled?

Example 9: Suppose $v(t) = t^2 - 4$ for $0 \leq t \leq 3$ describes a particle moving along a horizontal line where t is in seconds and $v(t)$ is in m/s. Find the

(a) displacement of the particle.

(b) distance traveled by the particle.