RECITATION 5: MIDTERM I REVIEW

PRACTICE PROBLEMS:

1. Sketch each of the functions below. Label all x- and y-intercepts and asymptotes. State, in interval notation, the domain and range of each function next to its graph.

(a) $y = 6 - x^4$

(b) $y = \sin(2x)$

(c) $y = \tan x$

(d) $y = \tan^{-1} x$

(e) $y = e^{x-1} + 2$

(f) $y = \ln x$

(g) $y = -2/(x + 3)$

(h) $y = \sqrt{x + 5}$

2. Evaluate the following limits. Show your work. *Make sure you are writing your mathematics correctly and clearly.*

(a) $\lim\limits_{t \to 2} \left(\dfrac{t^2 - 4}{t^3 - 3t + 5} \right)^3 =$

(b) $\lim\limits_{x \to 3} \dfrac{x^2 + 3x}{x^2 - x - 12} =$

(c) $\lim\limits_{x \to 4} \dfrac{x^2 - 4x}{x^2 - x - 12} =$

(d) $\lim\limits_{h \to 0} \dfrac{(h - 5)^2 - 25}{h} =$

3. For each function below, determine all the values in the domain of the function for which the function is continuous. (For the first example, you may find sketching the graph helpful, though you are not required to graph it.)

(a) $f(x) = \begin{cases} \frac{3}{x+5} & x < 1 \\ \frac{x+1}{2} & 1 \le x \le 3 \\ x^2 - 7 & 3 < x \end{cases}$

(b) $g(x) = \frac{2^x + 1}{\sqrt{1-x}}$

4. Find the limit or show that it does not exist. *Make sure you are writing your mathematics correctly and clearly.*

(a) $\displaystyle\lim_{x\to\infty} \frac{3x^4 + 2x}{\sqrt{x^4 + 2x}}$

(b) $\displaystyle\lim_{x\to\infty} \frac{\cos^2 x}{x^3 + 1}$

(c) $\displaystyle\lim_{x\to\infty} \frac{\sqrt[3]{8x^3 + 1}}{2 - 5x}$

5. Write the formula for a function with vertical asymptotes at $x = -1$ and $x = 3$ and a horizontal asymptote at $y = 4/3$.

6. State, formally, the definition of the derivative of a function $f(x)$ at $x = a$.

7. Let $f(x) = 5x^2 - 3x$.

    (a) Use the definition to find the derivative of $f(x)$.

    (b) Find the slope of the tangent line to $f(x)$ when $x = -3$.

    (c) Write the equation of the line tangent to $f(x)$ when $x = -3$.

8. Suppose N represents the number of people in the United States who travel by car to another state for a vacation this Memorial Day weekend when the average price of gasoline is p dollars per gallon. Would you expect dN/dp to be positive or negative? Explain your answer.

9. The graph of $f(x)$ is sketched below. On the same set of axes, give a rough sketch $f'(x)$.

t (minutes)
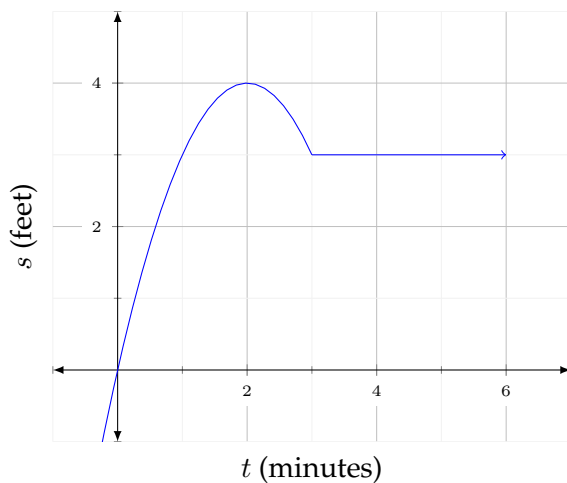
10. Find the domain of each function.

    (a) $f(x) = \sqrt{x^2 - x - 6}$

    (b) $g(t) = \ln(t + 6)$

11. Solve for x.

    (a) $e^{x-3} + 2 = 6$

    (c) $\ln x + \ln(x - 1) = 0$

    (b) $\ln(x + 5) - 3 = 7$

    (d) $\cos(8x) = 0$

12. Write the expression below as a single logarithm:

$$\frac{1}{3}\left(\ln(x + 2)^3\right) + \frac{2}{3}\left(\ln x - \ln(x^2 + 1)^2\right)$$

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13. Find the exact value of the following expressions.

(a) $\cos^{-1}(-1) =$ (b) $\sin^{-1}(0.5) =$ (c) $\tan^{-1}(\sqrt{3}) =$

14. Find the exact value of the following expressions:

(a) $\sin^{-1}(\sin(5\pi/4)) =$ (b) $\sin(\tan^{-1}x) =$

15. Evaluate $\lim\limits_{\theta \to \pi} \sec(\theta + \sin\theta)$