RECITATION: 3-1 TO 3-3 REVIEW OF BASIC DIFFERENTIATION

Disclaimer: On this quiz "Simplify" is short for "simplify your answer by combining like terms, factoring out any common factors and finding a common denominator, if necessary."

State the derivatives of the following functions:

- $\frac{d}{dx}x^n = \boxed{n x^{n-1}}$
- $\frac{d}{dx}e^x = \boxed{e^x}$
- $\frac{d}{dx}\sin x = \boxed{\cos x}$
- $\frac{d}{dx}\cos x = \boxed{-\sin x}$
- $\frac{d}{dx}\tan x = \boxed{\sec^2 x}$
- $\frac{d}{dx}\sec x = \boxed{\sec x \tan x}$
- $\frac{d}{dx}\csc x = \boxed{-\csc x \cot x}$
- $\frac{d}{dx}\cot x = \boxed{-\csc^2 x}$

Suppse f and g are differentiable functions. State the derivatives of the following functions. What rules are these?

Product Rule:

- $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule

- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

Example 1: Find the derivative of the following functions.

a) $y = \frac{1}{2}x^6 - 3x^4 + x$

$y' = \frac{1}{2}\cdot 6x^5 - 3\cdot 4x^3 + 1$

make sure you say you've taken the derivative.

$\boxed{y' = 3x^5 - 12x^3 + 1}$

b) $f(t) = 2 - \frac{2}{3}t + \tan x$

$f'(t) = 0 - \frac{2}{3} + \sec^2 x$

$\boxed{f'(t) = \sec^2 x - 2/3}$

Example 2: Find the derivative of the following functions.

a) $y = \pi^2 + \ln 2 + e^5$ ← these are all constants!

$y' = 0 + 0 + 0$

$\boxed{y' = 0}$

b) $f(x) = \sqrt[5]{x} + 4\sqrt{x^5} + \cot x$

$f(x) = x^{1/5} + 4x^{5/2} + \cot x$

$f'(x) = \frac{1}{5}x^{1/5-1} + 4\left(\frac{5}{2}\right)x^{5/2-1} - \csc^2 x$

$\boxed{f'(x) = \frac{1}{5x^{4/5}} + 10x^{3/2} - \csc^2 x}$

$= \boxed{\frac{1}{5\sqrt[5]{x^4}} + 10\sqrt{x^3} - \csc^2 x}$

Example 3: Find the derivative of the following functions.

a) $y = 5e^x + 3\cos x + \sec x$

$y' = 5e^x + 3(-\sin x) + \sec x \tan x$

$\boxed{y' = 5e^x - 3\sin x + \sec x \tan x}$

b) $y = 5 + 2\sin x + \sqrt{x}$

$y' = 0 + 2\cos x + \frac{1}{2}x^{-1/2}$

$\boxed{y' = 2\cos x + \frac{1}{2\sqrt{x}}}$

Example 4: Find the derivative of the following functions. Simplify.

a) $h(x) = (x^2+3)(x-5)$

simplify before taking derivative

$h(x) = x^3 - 5x^2 + 3x - 15$

$h'(x) = 3x^2 - 5 \cdot 2x + 3$

$\boxed{h'(x) = 3x^2 - 10x + 3}$

b) $y = \dfrac{x^3 - 2x + 6}{x^2}$

$y = \frac{x^3}{x^2} - \frac{2x}{x^2} + \frac{6}{x^2}$

$y = x - 2x^{-1} + 6x^{-2}$

$y' = 1 + 2x^{-2} - 12x^{-3}$

$y' = 1\frac{x^3}{x^3} + \frac{2}{x^2}\frac{x}{x} - \frac{12}{x^3}$

$\boxed{y' = \dfrac{x^3 + 2x - 12}{x^3}}$

Example 5: For what values of x does the graph of $f(x) = 2x^3 + 3x^2 - 12x + 1$ has a horizontal tangent?

Determine where $f'(x) = 0$

$6x^2 + 6x - 12 = 0$

$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$

$x + 2 = 0 \qquad x - 1 = 0$

$\boxed{x = -2} \qquad \boxed{x = 1}$

Example 6: Find the derivative of the following functions. Simplify.

a) $y = \dfrac{x+1}{x^3+x-2}$ Quotient Rule

$$y' = \frac{(x^3+x-2)1 - (x+1)(3x^2+1)}{(x^3+x-2)^2}$$

$$y' = \frac{x^3+x-2-(3x^3+x+3x^2+1)}{(x^3+x-2)^2}$$

$$\boxed{y' = \frac{-2x^3-3x^2-3}{(x^3+x-2)^2}}$$

b) $f(x) = x^3 \cos x$

$$f'(x) = 3x^2\cos x + x^3(-\sin x)$$

$$\boxed{f'(x) = x^2(3\cos x - x\sin x)}$$

Example 7: Suppose $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$. Find $h'(2)$ if $h(x) = \dfrac{g(x)}{2+f(x)}$.

$$h'(x) = \frac{(2+f(x))\,g'(x) - g(x)\,f'(x)}{(2+f(x))^2}$$

$$h'(2) = \frac{(2+f(2))\,g'(2) - g(2)\,f'(2)}{(2+f(2))^2}$$

$$= \frac{(2+(-3))(7) - 4(-2)}{(2+(-3))^2}$$

$$= \frac{(-1)(7)+8}{(-1)^2}$$

$$= \frac{1}{1}$$

$$= \boxed{1}$$

Example 8: For what values of x does $f(x) = x + 2\sin x$ have a horizontal tangent? (Hint: There are an infinite number of them. Don't give just one.)

find where $f'(x) = 0$!

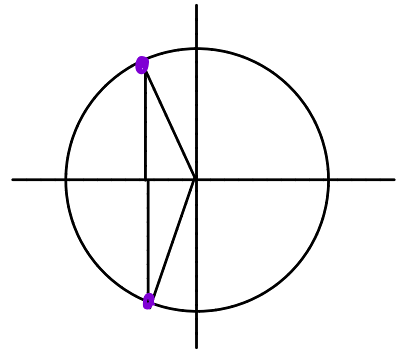
$f'(x) = 1 + 2\cos x$

$0 = 1 + 2\cos x$

$-1/2 = \cos x$

$\boxed{x = 2\pi/3 + 2\pi n}$

$\boxed{x = 4\pi/3 + 2\pi n}$



Example 9: Differentiate $f(\theta) = \theta\cos\theta\sin\theta$

$f'(\theta) = \left(\frac{d}{d\theta}\theta\right)\cos\theta\sin\theta + \theta\left(\frac{d}{d\theta}\cos\theta\right)\sin\theta + \theta\cos\theta\left(\frac{d}{d\theta}\sin\theta\right)$

$= 1\cos\theta\sin\theta + \theta(-\sin\theta)\sin\theta + \theta\cos\theta\cos\theta$

$= \boxed{\cos\theta\sin\theta - \theta\sin^2\theta + \theta\cos^2\theta}$

Example 10: Find an equation of the tangent line to $y = x + \tan x$ at (π, π). Give your answer in slope-intercept form.

① find slope! Find derivative, input value to get slope.

$y' = 1 + \sec^2 x$

$m = 1 + 1/(\cos\pi)^2 = 2$

② $y - y_1 = m(x - x_1)$

$y - \pi = 2(x - \pi)$

$y - \pi = 2x - 2\pi$

$\boxed{y = 2x - \pi}$