RECITATION: 3-4 TO 3-4 REVIEW OF BASIC DIFFERENTIATION

Disclaimer: On this quiz "Simplify" is short for "simplify your answer by combining like terms, factoring out any common factors and finding a common denominator, if necessary."

State the derivatives of the following functions:

- $\frac{d}{dx} b^x = \boxed{(\ln b)\, b^x}$
- $\frac{d}{dx} \sin^{-1} x = \boxed{1/\sqrt{1-x^2}}$
- $\frac{d}{dx} \cos^{-1} x = \boxed{-1/\sqrt{1-x^2}}$
- $\frac{d}{dx} \tan^{-1} x = \boxed{1/(1+x^2)}$

Suppse f and g are differentiable functions. State the derivatives of the following functions. What rules are these?

- $\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$
- $\frac{d}{dx} g(f(x)) = g'(f(x))\, f'(x)$

both the chain rule!

Example 1: Differentiate the following functions.

(a) $f(x) = e^{-x} \cos x$

$$f'(x) = e^{-x} \frac{d}{dx} \cos x + \frac{d}{dx} e^{-x} \cos x$$
$$= e^{-x}(-\sin x) + (-e^{-x}) \cos x$$
$$= \boxed{-e^{-x}(\sin x + \cos x)}$$

(b) $f(t) = \frac{\cot t}{e^{2t}}$

$$f'(t) = \frac{e^{2t}(-\csc^2 t) - \cot t\,(2e^{2t})}{(e^{2t})^2}$$
$$= \frac{-e^{2t}(\cot t + \csc^2 t)}{e^{4t}}$$
$$= \boxed{\frac{\cot t - \csc^2 t}{e^{2t}}}$$

Example 2: If $f(x) = \sec x$, find $f''(\pi/4)$.

$$f'(x) = \sec x \tan x$$
$$f''(x) = \underline{\sec x \tan x} \cdot \tan x + \sec x \cdot \underline{\sec^2 x}$$
$$= \boxed{\sec x\,(\tan^2 x + \sec^2 x)}$$
$$f''(\pi/4) = \sec(\pi/4)\,(\tan^2(\pi/4) + \sec^2(\pi/4))$$
$$= \sqrt{2}\,(1^2 + \sqrt{2}^2)$$
$$= \boxed{3\sqrt{2}}$$

note $\sec \pi/4 = \frac{1}{\cos\frac{\pi}{4}} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Example 3: Differentiate the following functions.

(a) $f(x) = \cos(x^2)$

$$f'(x) = -\sin(x^2) \cdot \frac{d}{dx} x^2$$
$$= \boxed{-2x \sin(x^2)}$$

(b) $f(x) = \sin^4(5x)$

$$= (\sin(5x))^4$$
$$f'(x) = 4(\sin(5x))^3 \frac{d}{dx} \sin(5x)$$
$$= 4\sin^3(5x)\cos(5x) \cdot 5$$
$$= \boxed{20\sin^3(5x)\cos(5x)}$$

Example 4: Differentiate the following functions.

(a) $y = 2^{x \tan x}$

product rule!

$$y' = (\ln 2)\, 2^{x\tan x}\left(\frac{d}{dx} x\tan x\right)$$
$$= (\ln 2)\, 2^{x\tan x}(1 \tan x + x\sec^2 x)$$
$$= \boxed{(\ln 2)\, 2^{x\tan x}(\tan x + x\sec^2 x)}$$

(b) $f(x) = \frac{1}{(1+\tan x)^2}$

$$= (1+\tan x)^{-2}$$
$$f'(x) = -2(1+\tan x)^{-3} \frac{d}{dx}(1+\tan x)$$
$$= \frac{-2}{(1+\tan x)^3} \cdot \sec^2 x$$
$$= \boxed{\frac{-2\sec^2 x}{(1+\tan x)^3}}$$

Example 5: Find the 50th derivative of $y = \cos(2x)$.

$$y = \cos(2x)$$
$$y' = -\sin(2x) \cdot 2 \quad 1$$
$$y'' = -\cos(2x) \cdot 2^2 \quad 2$$
$$y''' = \sin(2x) \cdot 2^3 \quad 3$$
$$y^{(4)} = \cos(2x)\, 2^4 \quad 4$$

$$\boxed{y^{(50)} = -2^{50}\cos(2x)}$$

$50 \div 4$ ← tells you how many 4 cycles

$$\begin{array}{r} 12 \\ 4\overline{)50} \\ -40 \\ \hline 10 \\ -8 \\ \hline 2 \end{array}$$

in group 2

product rule

Example 6: Given $x^2 - 4xy + y^2 = 4$ find dy/dx.

$$2x - 4\cdot y - 4x\cdot 1\,\frac{dy}{dx} + 2y\cdot\frac{dy}{dx} = 0$$

$$2x - 4y - 4x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} - 4x\frac{dy}{dx} = 4y - 2x$$

$$\frac{dy}{dx}(2y - 4x) = 4y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{4y-2x}{2y-4x} = \frac{2y-x}{y-2x}}$$

Example 7: Find an equation of the tangent line to $\sin(x+y) = 2x - 2y$ at the point (π, π).

$$\cos(x+y)\frac{d}{dx}(x+y) = 2 - 2\frac{dy}{dx}$$

you don't have to solve for dy/dx

$$\cos(x+y)\left(1 + \frac{dy}{dx}\right) = 2 - 2\,dy/dx$$

$$\cos(\pi+\pi)(1+m) = 2 - 2(m)$$

$$1(1+m) = 2 - 2m$$

$$1 + m = 2 - 2m$$

$$3m = 1$$

$$m = 1/3$$

$$y - y_1 = m(x - x_1)$$

$$y - \pi = \frac{1}{3}(x - \pi)$$

$$y = \frac{1}{3}x - \frac{\pi}{3} + \pi$$

$$\boxed{y = \frac{1}{3}x + \frac{2\pi}{3}}$$

Example 8: Differentiate the following functions.

(a) $y = \tan^{-1}(5x^3)$

$$y' = \frac{1}{1+(5x^3)^2}\cdot\frac{d}{dx}5x^3$$

$$= \boxed{\frac{15x^2}{1+25x^6}}$$

(b) $g(x) = \arccos(\sqrt{x})$

$$g'(x) = \frac{-1}{\sqrt{1-(\sqrt{x})^2}}\cdot\frac{d}{dx}\sqrt{x}$$

$$= \frac{-1}{\sqrt{1-x}}\cdot\frac{1}{2}x^{-1/2}$$

$$= \frac{-1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}$$

$$= \boxed{\frac{-1}{2\sqrt{x^2-x}}}$$

product rule

Example 9: Differentiate $y = \underline{x \sin^{-1} x} + \sqrt{1-x^2}$

$$y' = 1 \sin^{-1} x + x \cdot \frac{1}{\sqrt{1-x^2}} + \frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$$

$$= \boxed{\sin^{-1} x}$$

Example 10: Differentiate the following functions. Simplify.

(a) $f(x) = -3x^4 + \sqrt{x^5} + \pi^3 + e^7$

$$f(x) = -3x^4 + x^{5/2} + \pi^3 + e^7$$

$$f'(x) = -3 \cdot 4x^3 + \frac{5}{2} x^{5/2 - 1} + 0 + 0$$

$$\boxed{f'(x) = -12x^3 + \frac{5}{2} x^{3/2}}$$

(b) $y = \frac{x^2 - x + 2}{x}$

better to simplify first than to do the quotient rule.

$$y = \frac{x^2}{x} - \frac{x}{x} + \frac{2}{x}$$

$$y = x - 1 + 2x^{-1}$$

$$y' = 1 - 0 + 2(-1)x^{-2}$$

$$\boxed{y' = 1 - 2/x^2}$$

Example 11: Find y'' if $x^2 + y^2 = 1$.

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x\left(\frac{dy}{dx}\right)}{y^2}$$

input $\frac{dy}{dx}$

$$= \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

$$= \frac{\left(-y - x^2/y\right)\frac{y}{1}}{(y^2)\frac{y}{1}}$$

$$= \frac{(-y^2 - x^2)}{y^3}$$

$$= \frac{-(x^2+y^2)}{y^3}$$

$=1$ (see original)

$$= \boxed{-1/y^3}$$

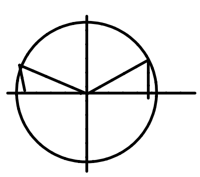
find where $y' = 0$

Example 12: Determine where the tangent line to $y = x + 2\cos x$ is horizontal.

$$y' = 1 - 2\sin x$$

$$0 = 1 - 2\sin x$$

$$\sin x = 1/2$$



$$\boxed{\begin{array}{l} x = \pi/6 + 2\pi n \\ x = 5\pi/6 + 2\pi n \end{array}}$$