Recitation 8: More Related Rates - Section 3-9

Directions: Set up and solve the ten related rates problems by following the strategy outlined in your text. Specifically, for each problem you should:

1. Read the problem carefully. Underline information you think will be needed in solving the problem.
2. Draw a diagram if possible. Pictures are very helpful!
3. Introduce notation and determine what rates you are given and what rate you need to find.
4. Write an equation that relates the various quantities of the problem. If necessary, use geometry to eliminate one variable by substitution. (We did this in the example about the conical tank in the notes.)
5. Use the Chain Rule to differentiate both sides of the equation with respect to t.
6. Substitute your known information and solve for the unknown rate of change. Then, summarize your findings in a complete sentence. Don't forget to include units with your final answer!

EX 1: A spherical balloon is being inflated at a rate of 2 in^3/min. Find the rate of increase of the radius with respect to the radius r when $r = 5$ inches. (Note: $V = \frac{4}{3}\pi r^3$)

① know $\frac{dV}{dt} = 2$ in^3/min; want dr/dt when $r = 5$.

② $\frac{d}{dt}(V) = \frac{d}{dt} \frac{4}{3}\pi r^3$

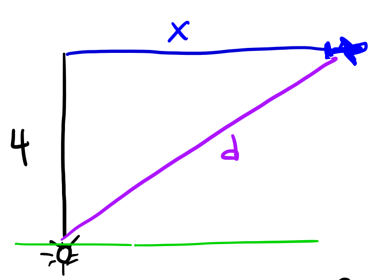
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

③ $2 = 4\pi(5^2)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{2}{100\pi}$ in/min

$= \boxed{\frac{1}{50\pi}\text{ in/min}}$

EX 2: A plane flying horizontally at an altitude of 4 miles and speed of 465 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 10 miles away from the station. Round the result to the nearest integer.



① know $dx/dt = 465$; want dd/dt when $d = 10$, note $4^2 + x^2 = 10^2 \Rightarrow 16 + x^2 = 100$

$\Rightarrow x^2 = 84 \Rightarrow x = \sqrt{84}$

$x = 2\sqrt{21}$

② $4^2 + x^2 = d^2$

$16 + x^2 = d^2$

③ $d/dt(16 + x^2) = d/dt\, d^2$

$2x\, dx/dt = 2d\, dd/dt$

$2\sqrt{21}(465) = 10(dd/dt)$

$\frac{dd}{dt} = \frac{\sqrt{21}\ 465}{5} = 93\sqrt{21} \approx \boxed{426\text{ mph}}$

EX 3: A snowball melts so that its surface area decreases at a rate of 4 cm^2/min. Find the rate at which the diameter decreases when the diameter is 40 cm. (Note: $A = 4\pi r^2$)

① Know $dA/dt = -4 \text{cm}^2/\text{min}$, want to know dr/dt when $r=20$.
(note dd/dt will be $2 * dr/dt$)

② $A = 4\pi r^2$

③ $\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

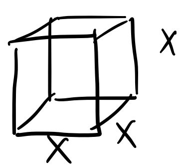
$-4 = 8\pi (20) \, dr/dt$

$dr/dt = \frac{-1}{40\pi}$ cm/min

④ Thus $dd/dt = -2/40\pi = \boxed{-1/20\pi \text{ cm/min}}$

EX 4: The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm?

this will require equations for volume and surface area.



① know $dV/dt = 10 \text{ cm}^3/\text{min}$.
want dA/dt when $x = 30$

② $V = x^3$ $\qquad A = 6x^2$

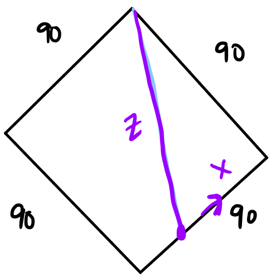
③ $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$ $\qquad \frac{dA}{dt} = 12x \, dx/dt$

$10 = 3(30)^2 \frac{dx}{dt}$ $\qquad = 12(30)(1/270)$

$10 = 3(900) \frac{dx}{dt}$ $\qquad = 12(3)/27$

$\frac{dx}{dt} = \frac{10}{2700} = \frac{1}{270}$ $\qquad = \boxed{4/3 \text{ cm}^2/\text{min}}$

EX 5: A baseball diamond is a square with sides of length 90 feet. A batter hits the ball and runs toward first base with a speed of 28 ft/sec. At what rate is his distance from second base decreasing when he is halfway to first base? Round the result to the nearest hundredth if necessary.



① Know $dx/dt = -28$ ft/sec, want dz/dt when $x = 45$

② $x^2 + 90^2 = z^2$ if $x = 45 \Rightarrow 45^2 + 90^2 = z^2$

$\Rightarrow 10125 = z^2$

$\Rightarrow z = \sqrt{10125}$

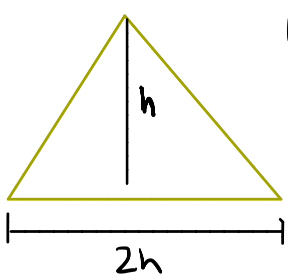
③ $\frac{d}{dt}(x^2 + 90^2) = \frac{d}{dt} z^2$

$$2x \frac{dx}{dt} = 2z \frac{dz}{dt}$$

$$45(-28) = \sqrt{10125} \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{-1260}{\sqrt{10125}} = \boxed{-12.512 \text{ ft/sec}}$$

EX 6: Gravel is being dumped from a conveyor belt at a rate of 35 ft^3/min and its coarseness is such that is forms a pile in the shape of a cone whose base diameter is twice the height. How fast is the height of the pile increasing when the pile is 15 feet high? Round your answer to the nearest hundredth. (Note $V = \frac{\pi}{3} r^2 h$)



① Know $dV/dt = 35$ ft^3/min ; want dh/dt when $h = 15$

② $V = \frac{\pi}{3} r^2 h$, note $r = h$ so

$V = \pi/3\, h^2 h \Rightarrow V = \pi/3\, h^3$

③ $\frac{d}{dt}(V) = \frac{d}{dt}(\pi/3\, h^3)$

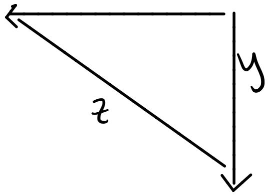
$$\frac{dV}{dt} = \pi h^2\, dh/dt$$

④ $35 = \pi (15^2)\, dh/dt$

$$\frac{35}{225\pi} = \frac{dh}{dt}$$

$$dh/dt = \boxed{7/45\pi \text{ ft/min}}$$

EX 7: Two cars start moving from the same point. One travels south at 28 mi/h and the other travels west at 70 mi/h. At what rate is the distance between the cars increasing 5 hours later? Round the result to the nearest hundredth.



① know $\frac{dx}{dt} = 70$, $\frac{dy}{dt} = 28$ mph

after 5 hours $x = 5(70) = 350$, $y = 5(28) = 140$

$z = \sqrt{350^2 + 140^2} = \sqrt{142,100} \approx 376.9615$

② $x^2 + y^2 = z^2$

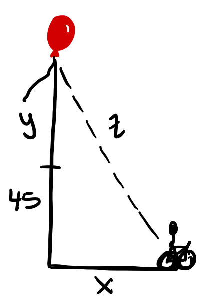
③ $\frac{d}{dt}(x^2+y^2) = \frac{d}{dt} z^2$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$

$350(70) + 140(28) = \sqrt{142,100}\ \frac{dz}{dt}$

④ $\frac{dz}{dt} = \frac{28,420}{\sqrt{142,100}} \approx \boxed{75.39 \text{ mph}}$

EX 8: A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 feet above him. How fast is the distance between the boy and the balloon increasing 3 s later?



① $dy/dt = 5$, $dx/dt = 15$ want dz/dt after 3 sec ⇒

$x = 3(15) = 45$, $y = 5(3) = 15$ and $z = \sqrt{45^2 + 60^2}$

$= 75$

② $x^2 + (45+y)^2 = z^2$

③ $\frac{d}{dt}(x^2 + (45+y)^2) = \frac{d}{dt} z^2$

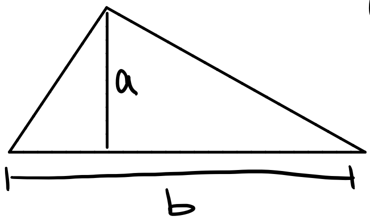
$2x\frac{dx}{dt} + 2(45+y)\frac{dy}{dt} = 2z\frac{dz}{dt}$

④ $45(15) + (45+15)(5) = 75\frac{dz}{dt}$

$\frac{975}{75} = \frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = 13 \text{ ft/sec}}$

EX 9: The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?



① know $da/dt = 1$, $dA/dt = 2$

want db/dt when $a = 10$, $A = 100$, so $A = \frac{1}{2}ab$

$100 = \frac{1}{2}(10) \cdot b$

$b = 20$

② $A = \frac{1}{2}ab$

③ $\frac{d}{dt} A = \frac{d}{dt} \frac{1}{2}ab$

$\frac{dA}{dt} = \frac{1}{2}\frac{da}{dt} b + \frac{1}{2} a \frac{db}{dt}$

④ $2 = \frac{1}{2}(1)(20) + \frac{1}{2}(10)\frac{db}{dt}$

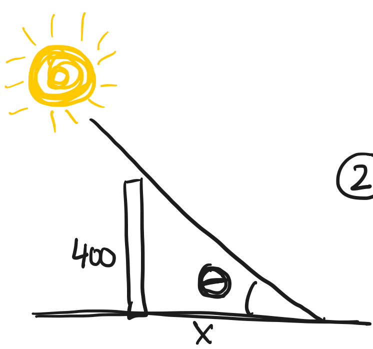
$2 = 10 + 5\, db/dt$

$-8 = 5\, db/dt$

$\boxed{db/dt = -8/5 \text{ cm/min}}$

EX 10: The angle of elevation of the sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400 foot tall building increasing when the angle of elevation of the sun is $\pi/6$ radians?

this one is easier!



① know $d\theta/dt = -0.25$ rad/hr

want dx/dt when $\theta = \pi/6$

② use either

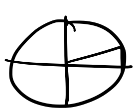
$\tan\theta = \frac{400}{x}$ or $\cot\theta = \frac{x}{400}$

③ $\frac{d}{dt}\cot\theta = \frac{d}{dt}\frac{x}{400}$

$-\csc^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{400}\frac{dx}{dt}$

$-\frac{400}{\sin^2\theta}\frac{d\theta}{dt} = \frac{dx}{dt}$

④ $\frac{dx}{dt} = \frac{-400}{(\sin(\pi/6))^2} \cdot -\frac{1}{4}$



$= \frac{100}{(1/2)^2}$

$= \boxed{400 \text{ ft/hr}}$