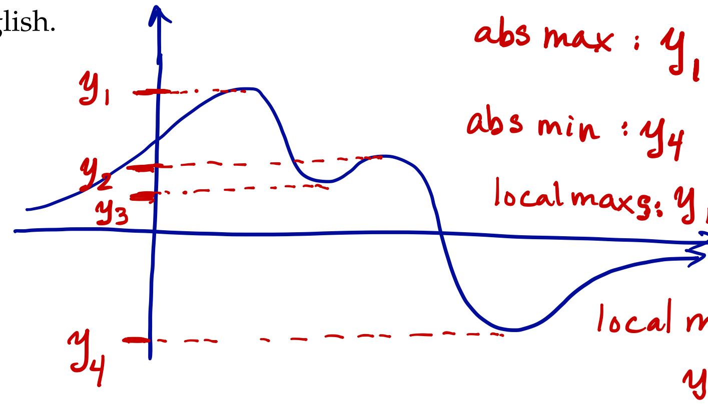
RECITATION 9: 1-4 MAXIMUM AND MINIMUM VALUES (PART 2)

WARM-UP QUESTIONS:

1. Explain the difference between an *absolute* minimum/maximum and a *local* minimum/maximum. Use pictures AND words in English.

An absolute max/min is the largest/smallest y-value possible for any x.

A local max/min is the largest/smallest y-value close by (or locally.)



abs max : y_1

abs min : y_4

local maxs: y_1, y_2

local mins: y_3, y_4

2. Are maximum and minimum values x-values or y-values?

y-values

3. Is it possible for a value to be *both* an absolute maximum and a local maximum? Explain.

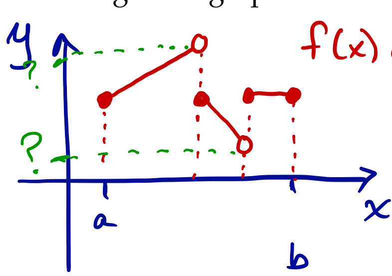
Yes. See y_1 above. If a y-value is largest EVERYWHERE, it is always largest LOCALLY!

4. State carefully and explicitly the TWO conditions used in the Extreme Value Theorem to ensure that a function, $f(x)$, is guaranteed to contain both an absolute minimum and an absolute maximum.

① – Gotta restrict the domain of f(x) to a closed interval, say [a,b]

② – f(x) must be continuous on [a,b]. No jumps. No holes. No vertical asymptotes.

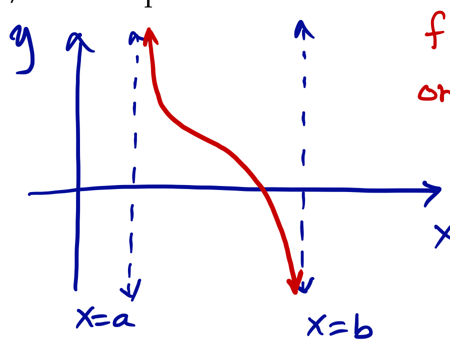
5. Draw pictures to show that BOTH conditions you stated above are required. That is, draw a picture of a graph with criteria 1 and *not* criteria 2 that fails to have a max or a min. Then draw a picture of a graph with criteria 2 and *not* critera 1 that fails to have a max or a min. Look at your neighbors graph and see if you agree with his/her examples.



f(x) on [a,b]

not continuous.

The "largest" and "smallest" values are never achieved!



f(x) is continuous on (a,b).

open interval.

6. *Very carefully* state what it means for $x = c$ to be a critical point of the function $f(x)$. **Hint:** There are sort of three "parts" to this definition.

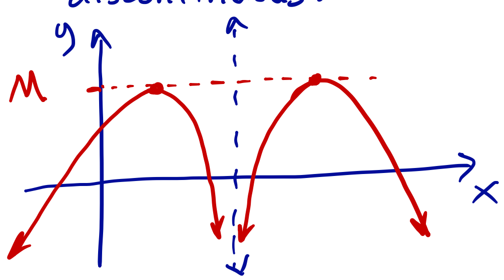
- $x=c$ has to in the domain of $f(x)$
- $f'(c)=0$ or $f'(c)$ is undefined

7. Again, *very carefully* explain **why** critical points are important to us (or, alternatively, explain what they can and cannot tell us about a function.)

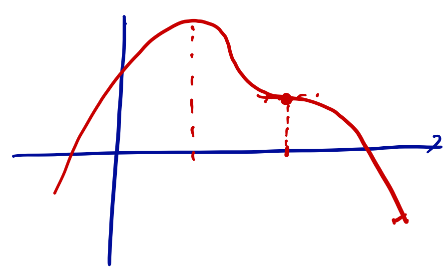
- Any local min/max on the interior of an interval must occur at a critical point. Thus, we look for maxs/mins here.
- If the domain has an end point, you must look here, too.
- Not every critical point is necessarily a max or min. You really must check.

8. Given the function $f(x)$, Tweedle Dee and Tweedle Dum together correctly find the derivative $f'(x)$ and they correctly find $f(x)$ as exactly two critical points. Tweedle Dee then immediately claims that one must be a local (and possibly an absolute) maximum and the other must be a local (and possibly absolute) minimum because it is not possible to have two maximums without a minimum inbetween. Tweedle Dum remains skeptical because their Calculus teacher ALWAYS makes them CHECK whether critical points are maximums or minimums and doesn't think the teacher would require that step if it wasn't necessary. Who is right and why?

- It is possible to have two local maximums if the function is discontinuous:



- If the function is continuous, it is possible to have a maximum and a "neither"



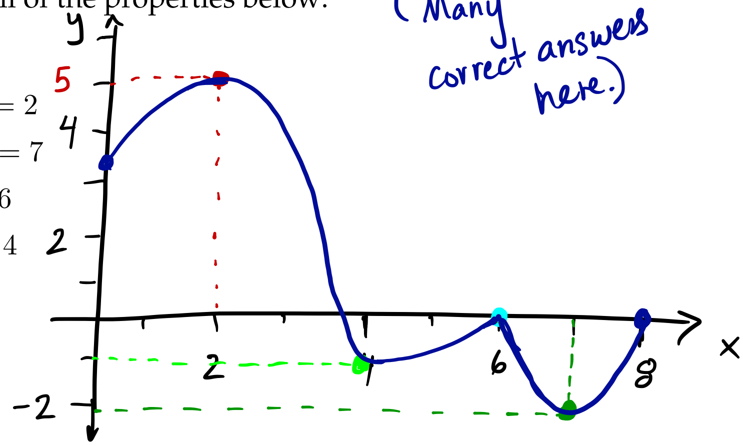
So Tweedle Dee is wrong.

PRACTICE PROBLEMS:

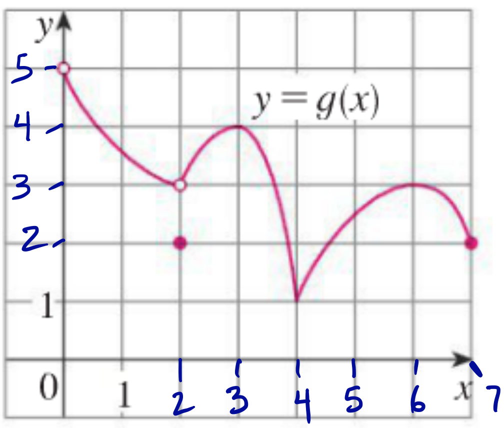
1. Sketch a graph of a function $f(x)$ with all of the properties below:

   (a) f is continuous on $[0, 8]$
   (b) f has absolute maximum of 5 at $x = 2$
   (c) f has absolute minimum of -2 at $x = 7$
   (d) f has a local maximum of 0 at $x = 6$
   (e) f has a local minimum of -1 at $x = 4$
   (f) f fails to be differentiable at $x = 6$

(Many correct answers here.)



2. State the absolute and local maximum and minimum values for $g(x)$ graphed below, if any exist. Also, identify where those extreme values occur.



local mins	$y=2$	$y=1$
location	$x=2$ and $x=7$	$x=4$

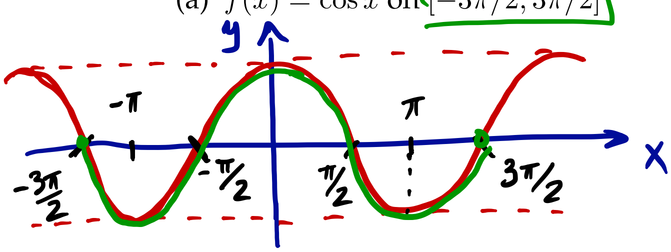
local maxs	$y=4$	$y=3$
location	$x=3$	$x=6$

absolute max: none

absolut min: $y=1$ at $x=4$

3. For each function below, sketch its graph and use the sketch to find the absolute and local maximum and minimum values of f.

   (a) $f(x) = \cos x$ on $[-3\pi/2, 3\pi/2]$

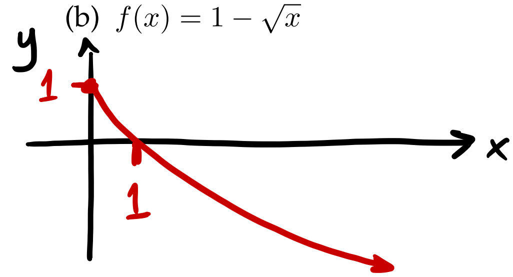


local maxs	$y=1$	$y=0$
location	$x=0$	$x=\frac{-3\pi}{2}$, $x=\frac{3\pi}{2}$

local mins	$y=-1$
location	$x=-\pi$ and $x=\pi$

absolute min: $y=-1$; absolute max: $y=1$

   (b) $f(x) = 1 - \sqrt{x}$



local max: $y=1$ at $x=0$

absolute max: $y=1$ at $x=0$

no local or absolute mins.

4. Find all critical numbers for the functions below.

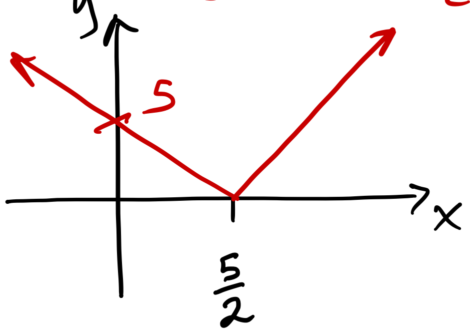
(a) $f(x) = |2x - 5|$

$2x - 5 = 0$ when $x = 5/2$

So $f'(x)$ is undefined at $x = 5/2$.

So critical pts: $x = \frac{5}{2}$

confirming graph sketch (rough)



(b) $h(p) = \frac{p-1}{p^2+4}$

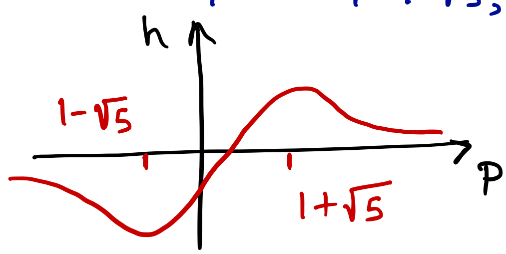
$$h'(p) = \frac{(p^2+4)(1) - (p-1)(2p)}{(p^2+4)^2} = \frac{-(p^2 - 2p - 4)}{(p^2+4)^2}$$

$h'(p)$ is defined for all p.

$h'(p) = 0$ when $p^2 - 2p - 4 = 0$ or $\frac{2 \pm \sqrt{4+16}}{2} = p$.

That is, $p = 1 \pm \sqrt{5}$

critical points $p = 1 + \sqrt{5}$, $p = 1 - \sqrt{5}$.



confirming rough sketch.

5. Find the absolute maximum and absolute minimum values of f on the given interval.

(a) $f(t) = (t^2 - 4)^3$ on $[-2, 3]$

$f'(t) = 3(t^2-4)^2 \cdot 2t = 6t(t^2-4)^2$

$f'(t)$ is defined everywhere.

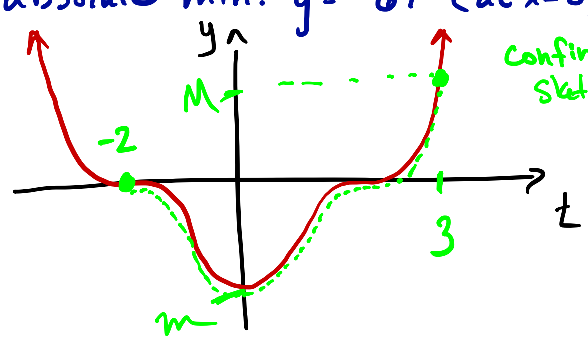
$f'(t) = 0$ when $t = 0, 2, -2$ critical pts

t	0	2	-2	3
$f(t)$	-64	0	0	125

end pts

absolute max: $y = 125$ (at $x = 3$)

absolute min: $y = -64$ (at $x = 0$)



confirming sketch

(b) $g(x) = x^{-2} \ln x$ on $[1/2, 4]$

$g'(x) = -2x^{-3} \cdot \ln x + x^{-2} \cdot \frac{1}{x} = \frac{-2\ln x + 1}{x^3}$

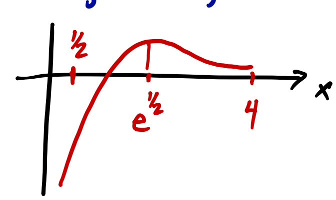
$g'(x)$ is defined for all x in $[\frac{1}{2}, 4]$

$g'(x) = 0$ if $-2\ln x + 1 = 0$ or $\ln x = \frac{1}{2}$ or $x = e^{1/2}$

x	$e^{1/2}$	$\frac{1}{2}$	4
$g(x)$	$(e^{1/2})^{-2} \cdot \ln(e^{1/2}) = \frac{1}{2e} \approx 0.184$	$(\frac{1}{2})^{-2} \cdot \ln(\frac{1}{2}) = 4\ln(\frac{1}{2}) \approx -2.77$	$\frac{\ln 4}{16} \approx 0.087$

absolute max: $y = 1/2e$; absolute min: $y = -4\ln 2$

confirming graph



6. After the consumption of an alcoholic beverage, the concentration of alcohol in the bloodstream (blood alcohol concentration or BAC) surges as the alcohol is absorbed, followed by a gradual decline as the alcohol is metabolized. The function

$$C(t) = 1.35te^{-2.802t}$$

models the average BAC, measured in mg/ML, of a group of eight male subjects t hours after rapid consumption of 15ML of ethanol (corresponding to one alcoholic drink).

(a) What is the maximum average BAC during the first 3 hours?

domain: $[0,3]$

$$c'(t) = 1.35\left(1\cdot e^{-2.802t} + t\cdot(-2.802)e^{-2.802t}\right)$$

$$= (1.35)\left(e^{-2.802t}\right)(1-2.802t)$$

$c'(t)$ is defined for all t in $[0,3]$

$c'(t)=0$ when $1-2.802t=0$ or $t=\frac{1}{2.802} \approx 0.357$ hours

t	0	0.357	3
$C(t)$	0	0.177	9.05×10^{-4}

absolute maximum : 0.177 mg/ML

(b) When does it occur? at 0.357 hours or 21.42 minutes

(c) confirming graph