RECITATION 9: 4-1 MAXIMUM AND MINIMUM VALUES (PART 2)

WARM-UP QUESTIONS:

1. Explain the difference between an *absolute* minimum/maximum and a *local* minimum/maximum. Use pictures AND words in English.

2. Are maximum and minimum values x-values or y-values?

3. Is it possible for a value to be *both* an absolute maximum and a local maximum? Explain.

4. State carefully and explicitly the TWO conditions used in the Extreme Value Theorem to ensure that a function, $f(x)$, is guaranteed to contain both an absolute minimum and an absolute maximum.

5. Draw pictures to show that BOTH conditions you stated above are required. That is, draw a picture of a graph with criteria 1 and *not* criteria 2 that fails to have a max or a min. Then draw a picture of a graph with criteria 2 and *not* critera 1 that fails to have a max or a min. Look at your neighbors graph and see if you agree with his/her examples.

6. *Very carefully* state what it means for $x = c$ to be a critical point of the function $f(x)$. **Hint:** There are sort of three "parts" to this definition.

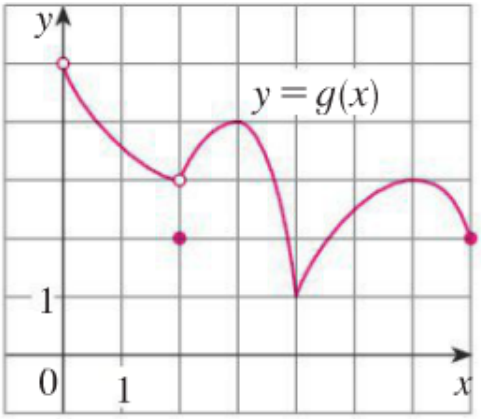
7. Again, *very carefully* explain **why** critical points are important to us (or, alternatively, explain what they can and cannot tell us about a function.)

8. Given the function $f(x)$, Tweedle Dee and Tweedle Dum together find the derivative $f'(x)$ and show $f(x)$ exactly two critical points one of which corresponds to a local maximum. Tweedle Dee then immediately claims that the other critical point must correspond to a local minimum because it is not possible to have two maximums without a minimum. Tweedle Dum remains skeptical because their Calculus teacher ALWAYS makes them CHECK whether critical points are maximums or minimums and doesn't think the teacher would require that step if it wasn't necessary. Who is right and why?

PRACTICE PROBLEMS:

1. Sketch a graph of a function $f(x)$ with all of the properties below:
   (a) f is continuous on $[0, 8]$
   (b) f has absolute maximum of 5 at $x = 2$
   (c) f has absolute minimum of -2 at $x = 7$
   (d) f has a local maximum of 0 at $x = 6$
   (e) f has a local minimum of -1 at $x = 4$
   (f) f fails to be differentiable at $x = 6$

2. State the absolute and local maximum and minimum values for $g(x)$ graphed below, if any exist. Also, identify where those extreme values occur.



3. For each function below, sketch its graph and use the sketch to find the absolute and local maximum and minimum values of f.
   (a) $f(x) = \cos x$ on $[-3\pi/2, 3\pi/2]$
   (b) $f(x) = 1 - \sqrt{x}$

4. Find all critical numbers for the functions below. Then confirm your answer is correct my making a rough sketch of the graph.

   (a) $f(x) = |2x - 5|$

   (b) $h(p) = \frac{p-1}{p^2+4}$

5. Find the absolute maximum and absolute minimum values of f on the given interval. Then confirm your answer is correct my making a rough sketch of the graph.

   (a) $f(t) = (t^2 - 4)^3$ on $[-2, 3]$

   (b) $g(x) = x^{-2} \ln x$ on $[1/2, 4]$

6. After the consumption of an alcoholic beverage, the concentration of alcohol in the bloodstream (blood alcohol concentration or BAC) surges as the alcohol is absorbed, followed by a gradual decline as the alcohol is metabolized. The function

$$C(t) = 1.35te^{-2.802t}$$

models the average BAC, measured in mg/ML, of a group of eight male subjects t hours after rapid consumption of 15ML of ethanol (corresponding to one alcoholic drink).

(a) What is the maximum average BAC during the first 3 hours?

(b) When does it occur?

(c) Confirm your answers to part (a) and part (b) using a rough sketch of the graph.