Recitation 3
More on Section 2-3: Calculating Limits Using the Limit Laws

Review: Complete the table below.

Limit Laws

In the rules below c is a constant, n is an integer, and $\lim_{x\to a} f(x)$ and $\lim_{x\to a}(x)$ both exist.

1. $\lim_{x\to a}[f(x)+g(x)] =$	2. $\lim_{x\to a}[f(x)-g(x)] =$
3. $\lim_{x\to a}[cf(x)] =$	4. $\lim_{x\to a}[f(x)\cdot g(x)] =$
5. $\lim_{x\to a}[f(x)/g(x)] =$	provided
6. $\lim_{x\to a}(f(x))^n =$	7. $\lim_{x\to a} c =$
8. $\lim_{x\to a} x =$	9. $\lim_{x\to a} x^n =$
10. $\lim_{x\to a} \sqrt[n]{x} =$	11. $\lim_{x\to a} \sqrt[n]{f(x)} =$
12. If $f(x)$ is a polynomial or rational function,	then $\lim_{x\to a} f(x) =$
13. The two-sided limit ($\lim_{x\to a} f(x)$) exists if and only if	

Goals:

- Many limits are evaluated by application of the limit laws above combined with a thoughtful use of *algebra*. We will practice this today.
- There remain limits too slippery for straightforward algebra. For this reason, we will learn a technique for finding limits using a bounding (or "squeezing") approach.
- We will also review the greatest integer function.

PRACTICE PROBLEMS (SET 1): Evaluate the following limits or explain why they do not exist.

1. $\lim_{x \to -2} \dfrac{x^3 + 2x^2 - 1}{5 - 3x}$

2. $\lim_{h \to 0} \dfrac{(h-5)^2 - 25}{h}$

3. (**hint:** rationalize the denominator.) $\lim_{t \to 0} \dfrac{t}{\sqrt{1+3t} - 1}$

4. $\lim_{z \to 1} \dfrac{8 - z}{c - z}$, where c is a constant.

5. $\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

6. $\lim_{x \to 0} \frac{\sqrt{16 - x} - 4}{x}$

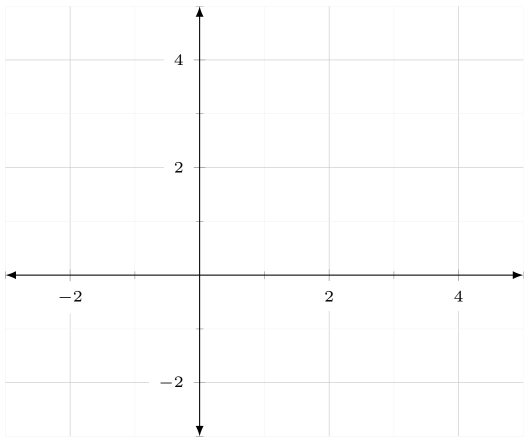
7. $\lim_{x \to 2} \frac{x^2 - 4}{2x^2 - 3x - 2}$

8. $\lim_{x \to 0} \frac{1}{x} - \frac{1}{|x|}$

PRACTICE PROBLEMS (SET 2): The Squeeze Theorem

1. Let $f(x) = \frac{1}{2}e^x$ and assume $g(x)$ is a function with the property that $g(x) \leq f(x)$ for all real numbers.

   (a) Graph and label $f(x)$ below.

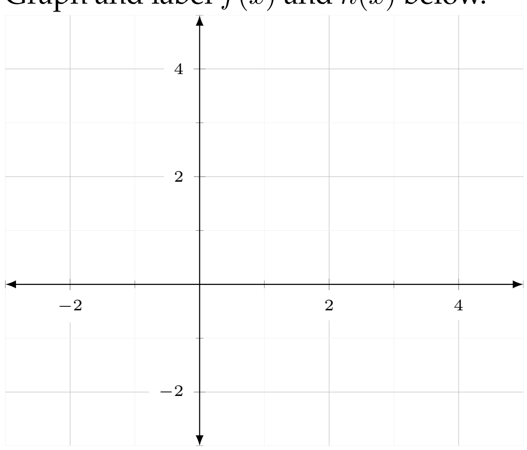
   

   (b) While we do not know exactly what $g(x)$ looks like, graph two *different* possible graphs of $g(x)$ and one example of a graph that could not possibly be $g(x)$. Compare your pictures with your neighbors' drawings.

   (c) Assume $\lim_{x\to 1} g(x) = L$, what can you say (if anything) about L?

2. Let $f(x) = x^2+2$ and let $h(x) = 2\cos x$. Assume that $g(x)$ is a function such that $h(x) \leq g(x) \leq f(x)$ for all real numbers.

   (a) Graph and label $f(x)$ and $h(x)$ below.

   (b) While we do not know exactly what $g(x)$ looks like, graph two *different* possible graphs of $g(x)$ and one example of a graph that could not possibly be $g(x)$. Compare your pictures with your neighbors' drawings.

   (c) What (if anything) can you say about $\lim_{x\to 0} g(x)$?

   (d) What (if anything) can you say about $\lim_{x\to \pi} g(x)$?

3. Fill in the blanks in the formal statement of the Squeeze Theorem:

   If $h(x) \leq g(x) \leq f(x)$ <u>for all real numbers</u> and $\lim_{x\to a} h(x) = L = \lim_{x\to a} f(x)$,

   then [ ]

4. Explain why you cannot evaluate $\lim_{\theta \to 0} \theta^2 \sin\left(\frac{1}{\theta}\right)$ by plugging in zero?

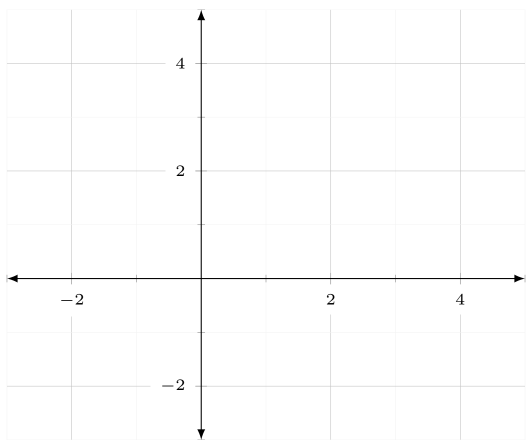
5. Use the Squeeze Theorem to evaluate the limit $\lim_{\theta \to 0} \theta^2 \sin\left(\frac{1}{\theta}\right)$.

PRACTICE PROBLEMS (SET 3): The Greatest Integer Function

Recall that the function $f(x) = [\![x]\!]$ is called *the greatest integer function* and outputs the greatest integer less than or equal to the input x. (Note that in other contexts this function is sometimes called *the floor function.*

1. Fill out the chart:

x	-2	-1	0	0.5	0.75	0.999	1	1.1	1.5	1.999	2
$[\![x]\!]$											



2. Sketch the graph of $f(x) = [\![x]\!]$:

3. Evaluate the limits below, if possible. If not, explain why they do not exist. Let n be an arbitrary integer.

(a) $\lim_{x \to 5^+} [\![x]\!]$

(b) $\lim_{x \to 5^-} [\![x]\!]$

(c) $\lim_{x \to 5} [\![x]\!]$

(d) $\lim_{x \to 5.5} [\![x]\!]$

(e) $\lim_{x \to 5.5} (3[\![x]\!] + \sqrt{2})$

(f) $\lim_{x \to n^+} [\![x]\!]$

(g) $\lim_{x \to n^-} [\![x]\!]$