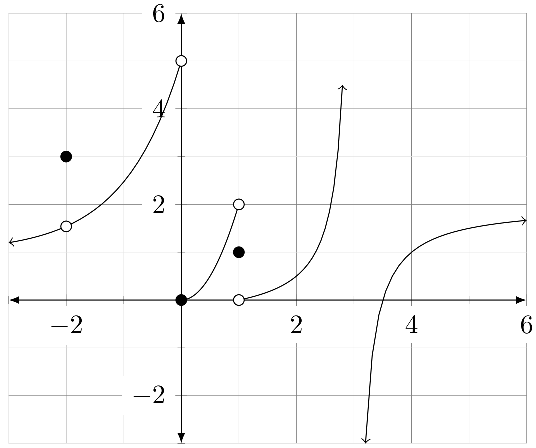
# Recitation 4
## Review of Sections 2.2-2.6

1. Use the graph of the function $f(x)$ to answer the questions below.




(a) $\lim_{x\to -2} f(x) =$ 1.5 $\qquad \lim_{x\to 0} f(x) =$ DNE

(b) $\lim_{x\to 1} f(x) =$ DNE $\qquad \lim_{x\to 2} f(x) =$ 0.5

(c) $\lim_{x\to 3} f(x) =$ DNE

(d) $\lim_{x\to 0^-} f(x) =$ 5 $\qquad \lim_{x\to 0^+} f(x) =$ 0

(e) $\lim_{x\to 3^-} f(x) =$ $\infty$ $\qquad \lim_{x\to 3^+} f(x) =$ $-\infty$

(f) $\lim_{x\to -\infty} f(x) =$ 1 $\qquad \lim_{x\to \infty} f(x) =$ 2

(g) $f(-2) =$ 3 $\qquad f(0) =$ 0

(h) $f(1) =$ 1 $\qquad f(3) =$ DNE

List all values for which $f(x)$ *fails* to be continuous.

$x = -2, 0, 1, 3$

List all asymptotes of $f(x)$ and identify which are vertical and which are horizontal.

vertical : $x=3$

horizontal: $x=1$, $x=2$

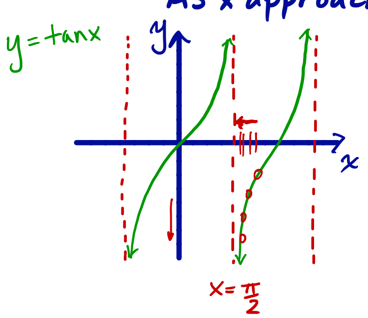
* 2. Evaluate the limits below: Justify your answer.

(a) $\lim_{x\to 3^-} \frac{\sqrt{x}}{(x-3)^5} = -\infty$

As x approaches 3 from below, $(x-3)^5$ approaches zero, also from below. So the denominator is always negative. The numerator approaches $\sqrt{3}$. Thus, the quotient approaches $-\infty$.

(b) $\lim_{x\to \frac{\pi}{2}^+} x \tan x = -\infty$

As x approaches $\frac{\pi}{2}$ from the right, tanx approaches $-\infty$. (See graph at left).




So $x \cdot \tan x$ approaches $-\infty$.

3. Evaluate the limits if they exist. If they do not exist, explain why.

(a) $\lim\limits_{x\to -2} \dfrac{x+2}{x^3+8} = \lim\limits_{x\to -2} \dfrac{(x+2)}{(x+2)(x^2-2x+4)}$

$= \lim\limits_{x\to -2} \dfrac{1}{x^2-2x+4} = \dfrac{1}{12}$

rationalize the numerator!

(b) $\lim\limits_{t\to 0} \dfrac{\sqrt{1+t}-\sqrt{1-t}}{t} \cdot \dfrac{\sqrt{1+t}+\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}} = \lim\limits_{t\to 0} \dfrac{1+t-(1-t)}{t(\sqrt{1+t}+\sqrt{1-t})}$

$= \lim\limits_{t\to 0} \dfrac{2t}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim\limits_{t\to 0} \dfrac{2}{\sqrt{1+t}+\sqrt{1-t}} = \dfrac{2}{1+1} = 1$

(c) $\lim\limits_{x\to -6} \dfrac{3x+18}{|x+6|} = \text{DNE}$

If $x \to -6^+$, then $|x+6| = x+6$. So $\dfrac{3x+18}{|x+6|} = \dfrac{3(x+6)}{x+6} = 3$.

If $x \to -6^-$, then $|x+6| = -(x+6)$. So $\dfrac{3x+18}{|x+6|} = \dfrac{3(x+6)}{-(x+6)} = -3$.

The left-hand limit and right-hand limit are not equal.

numerical reasoning and algebra

4. Find the value of $c$ such that $B(t)$ is a continuous function where $B(t) = \begin{cases} 4-\frac{1}{2}t & t<2 \\ \sqrt{t+c} & t \geq 2. \end{cases}$

We know $\lim\limits_{t\to 2^-} B(t) = \lim\limits_{t\to 2^-} 4-\frac{1}{2}t = 4-1=3.$

and

$\lim\limits_{t\to 2^+} B(t) = \lim\limits_{t\to 2^+} \sqrt{t+c} = \sqrt{2+c}$.

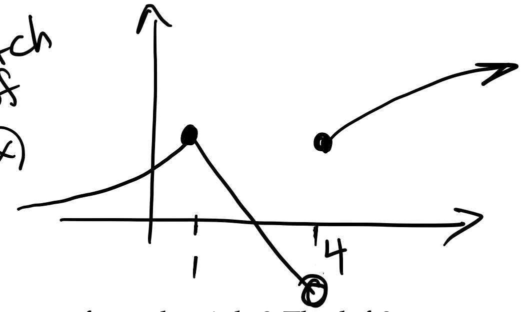
In order to be continuous, we need $\lim\limits_{t\to 2^-} B(t) = \lim\limits_{t\to 2^+} B(t)$.

That is, we require $3 = \sqrt{2+c}$.

Thus, $9 = 2+c$ and $\boxed{c=7}$

Do you understand why I used different expressions here?

rough Sketch of $f(x)$




5. Given $f(x) = \begin{cases} 2^x & x \le 1 \\ 3 - x & 1 < x \le 4 \\ \sqrt{x} & 4 < x, \end{cases}$

(a) find all the numbers at which $f$ is discontinuous.

(b) Of the numbers from part (a), at which is $f(x)$ continuous from the right? The left?

(a) at $x=1$:

$\lim_{x\to 1^-} f(x) = 2^1 = 2$

$\lim_{x\to 1^+} f(x) = 3-1 = 2$

thems is equal

at $x=4$:

$\lim_{x\to 4^-} f(x) = 3-4 = -1$

$\lim_{x\to 4^+} f(x) = \sqrt{4} = 2$

not equal!

answer to part (a): $f(x)$ is discontinuous at $x=4$

(b.) $f(4) = 3-4 = -1$ and so $f(4) = \lim_{x\to 4^-} f(x)$.

Answer: At $x=4$, $f(x)$ is continuous from the left.

6. State the Intermediate Value Theorem and draw the associated picture.

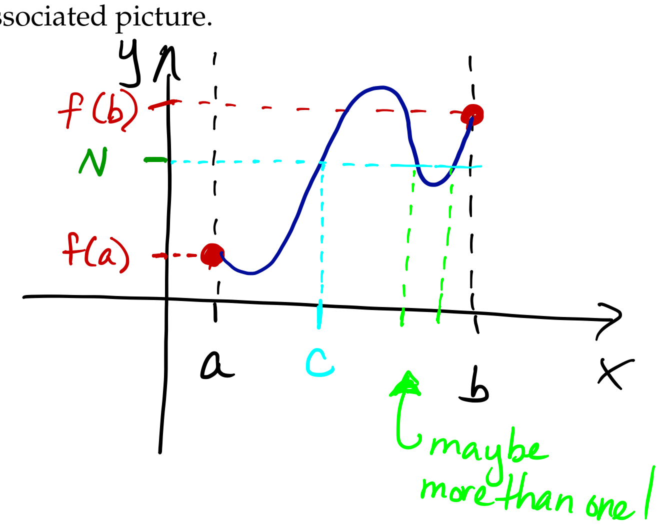
If • $f$ is conts on $[a,b]$,

• $f(a) \neq f(b)$,

• $N$ is between $f(a)$ and $f(b)$

then there is an x-value $c$ in $(a,b)$ so that

$f(c) = N$.




7. Use the Intermediate Value Theorem to show that the equation $\sin x = x^2 - x$ must have a solution in the interval $(1,2)$.

thinking:

pick $f(x) = x^2 - x - \sin x$

$a=1$, $b=2$

$f(a) = 1^2 - 1 - \sin(1) = -\sin(1) < 0$ because $1 < \pi$.

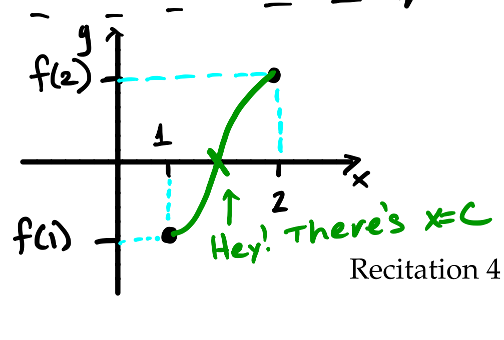
$f(b) = f(2) = 4 - 2 - \sin(2) = 2 - \sin(2) > 0$ because $\sin\theta < 1$ always.

$f$ is continuous because $x^2$, $x$, and $\sin x$ are!

Answer:

Since $f(x) = x^2 - x - \sin x$ is continuous on $[1,2]$, $f(1) < 0$, and $f(2) > 0$, the Intermediate Value Theorem says there is some c-value in $(1,2)$ so that $f(c) = 0$. So $x=c$ is a solution to the equation.

Aside: picture I have in my head.



8. For each of the following, find the limit or show that it does not exist.

(a) $\displaystyle\lim_{x\to-\infty} \frac{4x^3-5x^2-3}{\sqrt{3}x^3+x+\pi}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}$

$$= \lim_{x\to-\infty}\frac{4-5/x-3/x^3}{\sqrt{3}+1/x^2+\pi/x^3}$$

$$=\frac{4+0+0}{\sqrt{3}+0+0}=4/\sqrt{3}$$

(b) $\displaystyle\lim_{x\to\infty}\frac{\sqrt{2+5x^6}}{4+x^3}\cdot\frac{1/x^3}{1/x^3} = \lim_{x\to\infty}\frac{\sqrt{\frac{2}{x^3}+5}}{\frac{4}{x^3}+1}$

use $x^3=\sqrt{x^6}$ for $x>0$

$$=\frac{\sqrt{5}}{1}=\sqrt{5}$$

use $\frac{1}{x^3}\to 0$ as $x\to\infty$.

tricky!

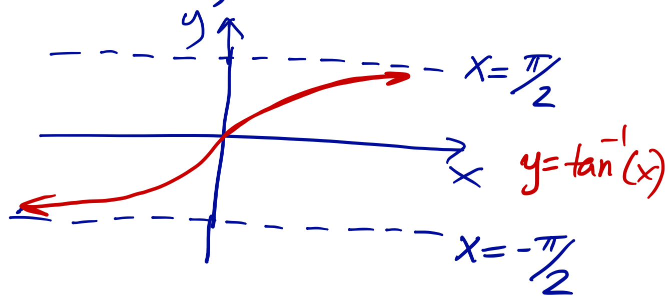
(c) $\displaystyle\lim_{x\to-\infty}(\sqrt{9x^2+4x}-3x) = \infty$

As $x\to-\infty$, $-3x$ approaches $+\infty$ and $\sqrt{9x^2+4}$ approaches $+\infty$. So their sum approaches $+\infty$.

(d) $\displaystyle\lim_{x\to 0^+}\tan^{-1}(\ln x) = -\frac{\pi}{2}$

As $x\to 0^+$, $\ln x$ approaches $-\infty$.

As $z\to-\infty$, $\tan^{-1}(z)$ approaches $-\frac{\pi}{2}$.




9. Find the horizontal and vertical asymptotes, if any.

(a) $f(x)=\frac{4+8x}{3x-1}$

as $x\to\pm\infty$, $f(x)=\frac{8}{3}$.

horizontal asymptote: $y=8/3$

as $x\to\frac{1}{3}^+$, $f(x)\to\infty$.

vertical asymptote: $x=\frac{1}{3}$

(b) $g(t)=\frac{t^3-t}{t^2-6t+5} = \frac{t(t^2-1)}{(t-5)(t-1)} = \frac{t(t+1)(t-1)}{(t-5)(t-1)}$

as $t\to\pm\infty$, $g(t)\to\pm\infty$.

So no horizontal asymptotes.

From the factored form, we see $t=1$ is where a removable discontinuity occurs.

(i.e. as $t\to 1$, $g(t)\to -1/2$.)

as $t\to 5^+$, $g(t)\to\infty$.

Thus a vertical asymptote at $\boxed{x=5.}$