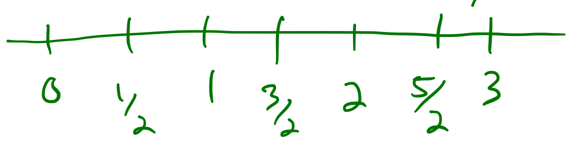
A calculator will be useful for this activity.

1. A radar gun was used to record the speed of a runner at the times given in the table.

t (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
$v(t)$ (m/s)	0	4.6	7.3	8.9	9.7	10.2	10.5

(a) Draw the t-axis with labels when the interval $[0, 3]$ is broken into $n = 6$ subintervals.

$\Delta t = \frac{3-0}{6} = \frac{1}{2}$



(b) Estimate the distance the runner covered during the 3 seconds using a left Riemann sum. (I.e., L_6)

$L_6 = \frac{1}{2}\left(v(0) + v(\tfrac{1}{2}) + v(1) + v(\tfrac{3}{2}) + v(2) + v(\tfrac{5}{2})\right)$

$= \frac{1}{2}(0 + 4.6 + 7.3 + 8.9 + 9.7 + 10.2)$

$= \boxed{20.35\text{ m}}$

(c) Estimate the distance the runner covered during the 3 seconds using a right Riemann sum.

$R_6 = \frac{1}{2}(4.6 + 7.3 + 8.9 + 9.7 + 10.2 + 10.5)$

$= \boxed{25.6\text{ m}}$

(d) Estimate the distance the runner covered during the 3 seconds using midpoints. [Hint: you can't use 6 subintervals as you don't have the function values at the midpoints of each subinterval. Try three instead]

$\Delta x = 1$

$M_3 = 1(4.6 + 8.9 + 10.2)$

$= \boxed{23.7\text{ m}}$

2. Let's evaluate $\int_0^2 3x\,dx$ by definition. The summation formula (*) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ will come in handy.

(a) Find an expression for R_n, an approximation of the area under $f(x) = 3x$ on $[0, 2]$ with right endpoints and n subintervals. [Hint: $x_1 = 2/n$, $x_2 = (2 \cdot 2)/n$, $x_3 = (3 \cdot 2)/n$,...] You will start with an expression in summation notation and use the given formula (*) to write the expression without summation.

$$R_n = \frac{2}{n}\left(f\left(\frac{2}{n}\right) + f\left(\frac{2\cdot 2}{n}\right) + f\left(\frac{3\cdot 2}{n}\right) + \dots + f\left(\frac{n\cdot 2}{n}\right)\right) \quad \Delta x = \frac{2-0}{n} = \frac{2}{n}$$

$$= \frac{2}{n}\left(3\left(\frac{2}{n}\right) + 3\left(\frac{2\cdot 2}{n}\right) + 3\left(\frac{3\cdot 2}{n}\right) + \dots + 3\left(\frac{n\cdot 2}{n}\right)\right)$$

$$= \frac{2}{n}\sum_{i=1}^{n} 3\left(\frac{2i}{n}\right)$$

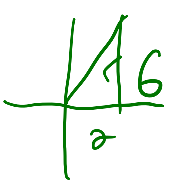
$$= 3\left(\frac{2}{n}\right)^2 \sum_{i=1}^{n} i = \frac{12}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$\boxed{R_n = \frac{6n(n+1)}{n^2}}$$

(b) Find the exact area under $f(x) = 3x$ on $[0, 2]$ by evaluating $\lim_{n\to\infty} R_n$.

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{6n^2 + 6n}{n^2} = \boxed{6 = A}$$

(c) Check that the area you computed is correct using geometry.



triangle

$$A = \frac{1}{2}(2)(6) = \boxed{6}$$

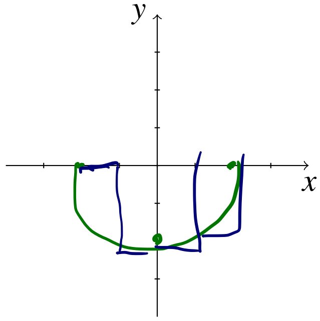
(d) Check that the area you computed is correct using FTC [part II].

$$\int_0^2 3x\,dx = \frac{3}{2}x^2\Big|_0^2 = \frac{3}{2}(2)^2 - \frac{3}{2}(0)^2$$

$$= \frac{12}{2} = \boxed{6}$$

3. Consider $\int_{-2}^{2} -\sqrt{4-x^2}\,dx$.

(a) Draw a graph depicting the area described by the integral.



$y = -\sqrt{4-x^2}$

$x^2 + y^2 = 4 \leftarrow$ but just bottom half

(b) Estimate the area described by the integral using left endpoints and $n = 4$.

$\Delta x = \frac{2-(-2)}{4} = 1$

$L_4 = 1\left(-\sqrt{4-(-2)^2} - \sqrt{4-(-1)^2} - \sqrt{4-0^2} - \sqrt{4-1^2}\right)$

$= 1\left(0 - \sqrt{3} - 2 - \sqrt{3}\right) = \boxed{-2-2\sqrt{3}}$

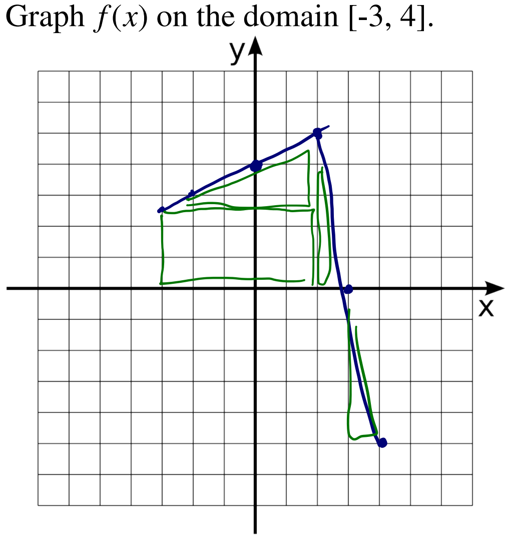
(c) Evaluate the integral exactly using a known geometric formula.

semicircle ; $r = 2$

$A = \frac{1}{2}\pi(2)^2 = \frac{1}{2}4\pi = 2\pi$ but $\boxed{A = -2\pi}$

4. Let $f(x) = \begin{cases} \frac{1}{2}x + 4, & x < 2 \\ 15 - 5x, & x \geq 2 \end{cases}$

(a) Graph $f(x)$ on the domain [-3, 4].



(b) Use known geometric formulas to evaluate $\int_{-3}^{4} f(x)\,dx$.

$= 5(2.5) + \frac{1}{2}(5)(2.5)$

$+ \cancel{\frac{1}{2}(1)(5)} - \cancel{\frac{1}{2}(1)(5)}$

$= \frac{25}{2} + \frac{25}{4} = \boxed{\frac{75}{4} \text{ or } 18.75}$

5. Is it true that $\int_a^b [f(x)]^2\,dx = \int_a^b f(x)\,dx \cdot \int_a^b f(x)\,dx$? Before answering, evaluate (a) - (d) using FTC [part II] or geometry:

(a) $\int_2^4 x\,dx$ $= \frac{1}{2}x^2\Big|_2^4 = \frac{1}{2}(4^2-2^2)$

$= \frac{1}{2}(16-4) = \boxed{6}$

or it's a trapezoid!

(b) $\int_2^4 x^2\,dx$ $= \frac{1}{3}x^3\Big|_2^4 = \frac{1}{3}(4^3-2^3)$

$= \frac{1}{3}(64-8) = \boxed{\frac{56}{3}}$ (not 6^2)

(c) $\int_0^{\pi/2} \cos x\,dx$, [note $\int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4}$]

$= \sin x\Big|_0^{\pi/2} = \sin \pi/2 - \sin 0 = 1$ (and $1^2 \neq \pi/4$)

(d) $\int_0^1 2\,dx$ and $\int_0^1 4\,dx$

$\int_0^1 2\,dx = 2(1-0) = 2$ $\int_0^1 4\,dx = 4(1-0) = 4$

hmm... $2^2 = 4$!!

(e) Answer the question posed at the beginning of the page.

Not necessarily. I.e. not for a generic function and interval $f(x)$, $[a,b]$.

6. Suppose we have a function $f(x)$ such that:

- $\int_{-2}^7 f(x)\,dx = -18$,
- $\int_{-2}^6 f(x)\,dx = 7$,
- $\int_5^7 f(x)\,dx = 8$.

Find $\int_5^6 f(x)\,dx$.

overlap! So $\int_5^6 f(x)$ is counting twice!

$\int_5^6 f(x) + \int_{-2}^7 f(x)\,dx = \int_{-2}^5 f(x)\,dx + \int_5^6 f(x)\,dx + \int_5^6 f(x)\,dx + \int_6^7 f(x)\,dx$

$A \quad - 18 \quad = \quad 7 \quad + \quad 8$ $\boxed{A = 33}$