A calculator will be useful for this activity.

1. A radar gun was used to record the speed of a runner at the times given in the table.

t (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
$v(t)$ (m/s)	0	4.6	7.3	8.9	9.7	10.2	10.5

(a) Draw the t-axis with labels when the interval $[0, 3]$ is broken into $n = 6$ subintervals.

(b) Estimate the distance the runner covered during the 3 seconds using a left Riemann sum. (I.e., L_6)

(c) Estimate the distance the runner covered during the 3 seconds using a right Riemann sum.

(d) Estimate the distance the runner covered during the 3 seconds using midpoints. [Hint: you can't use 6 subintervals as you don't have the function values at the midpoints of each subinterval. Try three instead]

2. Let's evaluate $\int_0^2 3x\,dx$ by definition. The summation formula (*) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ will come in handy.

(a) Find an expression for R_n, an approximation of the area under $f(x) = 3x$ on $[0, 2]$ with right endpoints and n subintervals. [Hint: $x_1 = 2/n$, $x_2 = (2 \cdot 2)/n$, $x_3 = (3 \cdot 2)/n$,...] You will start with an expression in summation notation and use the given formula (*) to write the expression without summation.

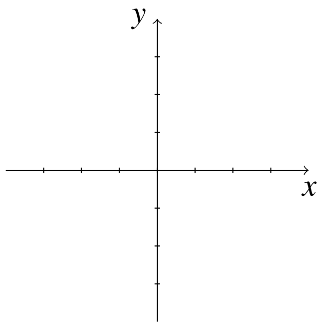
(b) Find the exact area under $f(x) = 3x$ on $[0, 2]$ by evaluating $\lim_{n\to\infty} R_n$.

(c) Check that the area you computed is correct using geometry.

(d) Check that the area you computed is correct using FTC [part II].

3. Consider $\int_{-2}^{2} -\sqrt{4-x^2}\,dx$.

(a) Draw a graph depicting the area described by the integral.

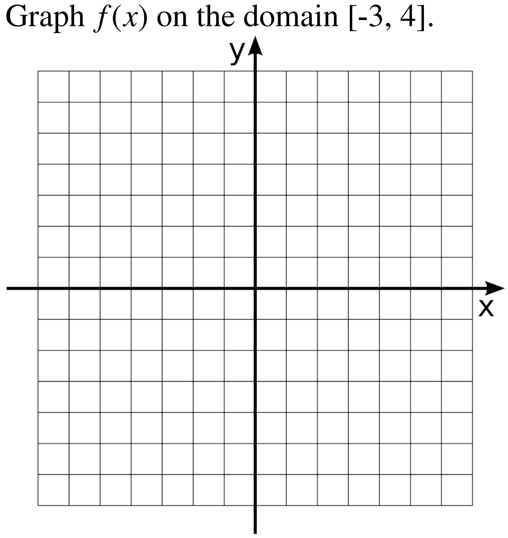


(b) Estimate the area described by the integral using left endpoints and $n = 4$.

(c) Evaluate the integral exactly using a known geometric formula.

4. Let $f(x) = \begin{cases} \frac{1}{2}x + 4, & x < 2 \\ 15 - 5x, & x \geq 2 \end{cases}$

(a) Graph $f(x)$ on the domain [-3, 4].



(b) Use known geometric formulas to evaluate $\int_{-3}^{4} f(x)\,dx$.

5. Is it true that $\int_a^b [f(x)]^2\,dx = \int_a^b f(x)\,dx \cdot \int_a^b f(x)\,dx$? Before answering, evaluate (a) - (d) using FTC [part II] or geometry:

(a) $\int_2^4 x\,dx$

(b) $\int_2^4 x^2\,dx$

(c) $\int_0^{\pi/2} \cos x\,dx$, [note $\int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4}$]

(d) $\int_0^1 2\,dx$ and $\int_0^1 4\,dx$

(e) Answer the question posed at the beginning of the page.

6. Suppose we have a function $f(x)$ such that:

- $\int_{-2}^7 f(x)\,dx = -18$,
- $\int_{-2}^6 f(x)\,dx = 7$,
- $\int_5^7 f(x)\,dx = 8$.

Find $\int_5^6 f(x)\,dx$.