

1. Consider the function $f(x)$ and its derivatives:

$$f(x) = \frac{e^x}{1+x}$$

$$f'(x) = \frac{xe^x}{(1+x)^2}$$

$$f''(x) = \frac{e^x(x^2+1)}{(1+x)^3}.$$

- a. Find the critical numbers of $f(x)$.
- b. Find the open intervals on which the function is increasing or decreasing.
- c. Find the open intervals on which the function is concave up or concave down.

d. Classify all critical points – using the first derivative test.

e. Classify all critical points – using the second derivative test.

f. Find the inflection points.

g. Sketch the graph.

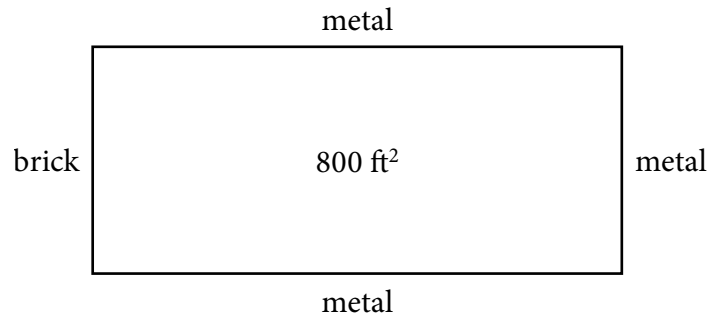
4. A ball of metal is being heated in an oven, and its radius is increasing at a rate of 0.1 cm/min. At what rate is the ball's volume increasing when its radius is 3 cm?

5. Evaluate the following limits.

$$\lim_{x \rightarrow 0} \frac{1 + x - e^x}{\sin x}$$

$$\lim_{x \rightarrow 0^+} (1 + 2x)^{1/x}$$

6. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$30 per foot and on the other three sides with a metal fence costing \$10 per foot. The area of the garden is to be 800ft^2 . What are the dimensions of the garden that minimize the cost of the fencing?



7.

a. State the Mean Value Theorem and draw a picture to illustrate it.

b. Suppose $f(x)$ is continuous on $[-1, 1]$ and has a derivative at each x in $(-1, 1)$. If $f(-1) = 7$ and $f(1) = 5$, what does the Mean Value Theorem let you conclude?