Solutions

LECTURE: 1-3: TRANSFORMATIONS AND TRIGONOMETRY REVIEW

Transformation Review

1. Explain what each does to the *original* graph $y = f(x)$. (Assume $c > 0$.)

(a) $f(x) + c$ — c units up

(b) $f(x) - c$ — down

(c) $\boxed{f(x + c) \quad \text{left}}$ — to memorize or to think?? The "old" f(0) is now found when $\underline{\underline{x=-c}}$.

(d) $f(x - c)$ — right

(e) $cf(x)$ — vertical stretch/shrink ($c>1$ / $0<c<1$)

(f) $f(cx)$ — horizontal stretch/shrink ($0<c<1$ / $c>1$)

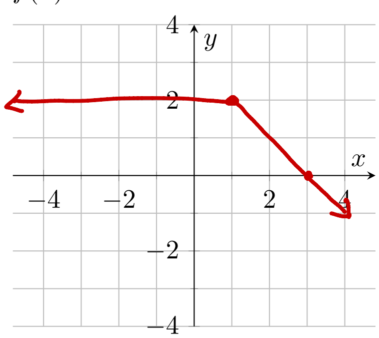
(g) $-f(x)$ — reflect about x-axis

(h) $f(-x)$ — reflect about y-axis

2. Let $f(x) = \begin{cases} 2 & x \leq 1 \\ 3 - x & x > 1 \end{cases}$. Graph each of the following *using the ideas from # 1 above.*

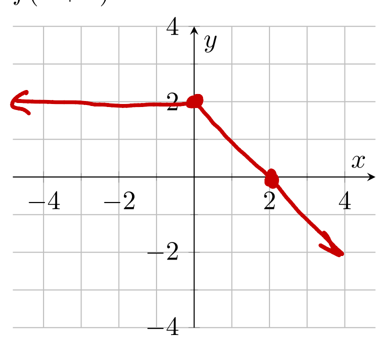
piecewise

(a) $f(x)$



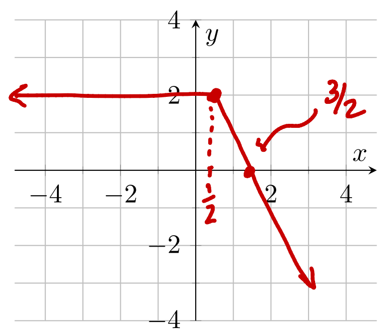
(b) $f(x + 1)$

Shift left 1 unit



(c) $f(2x)$

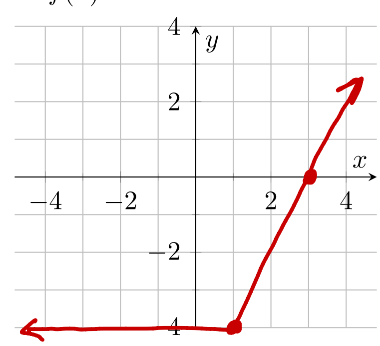
Shrink by factor of 2 horizontally



(d) $-2f(x)$

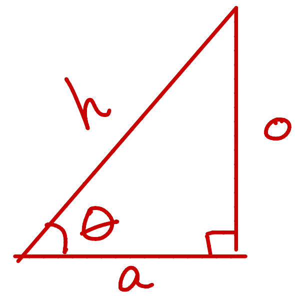
• reflect about x-axis

• stretch vertically by factor of 2



Three Views of Trigonometric Functions

- sides of a right triangle
- points on the unit circle
- graphs in the xy-plane

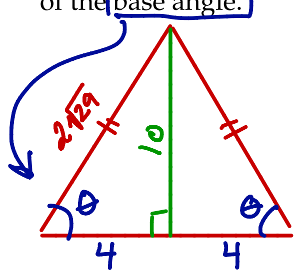


The Triangle Defintion

3. Sketch a right triangle with side a adjacent to an angle θ, o opposite of the angle θ and hypotenuse h. Define each of the six trigonometric functions in terms of that triangle.

a) $\sin\theta$ $\frac{o}{h}$ b) $\cos\theta$ $\frac{a}{h}$ c) $\tan\theta$ $\frac{o}{a}$ d) $\sec\theta$ $\frac{1}{\cos\theta}$ $\frac{h}{a}$ e) $\csc\theta$ $\frac{1}{\sin\theta}$ $\frac{h}{o}$ f) $\cot\theta$ $\frac{1}{\tan\theta}$ $\frac{a}{o}$

4. An isosceles triangle has a height of 10 ft and its base is 8 feet long. Determine the sine, cosine and tangent of the base angle.



$\tan\theta = \frac{10}{4} = \frac{5}{2}$

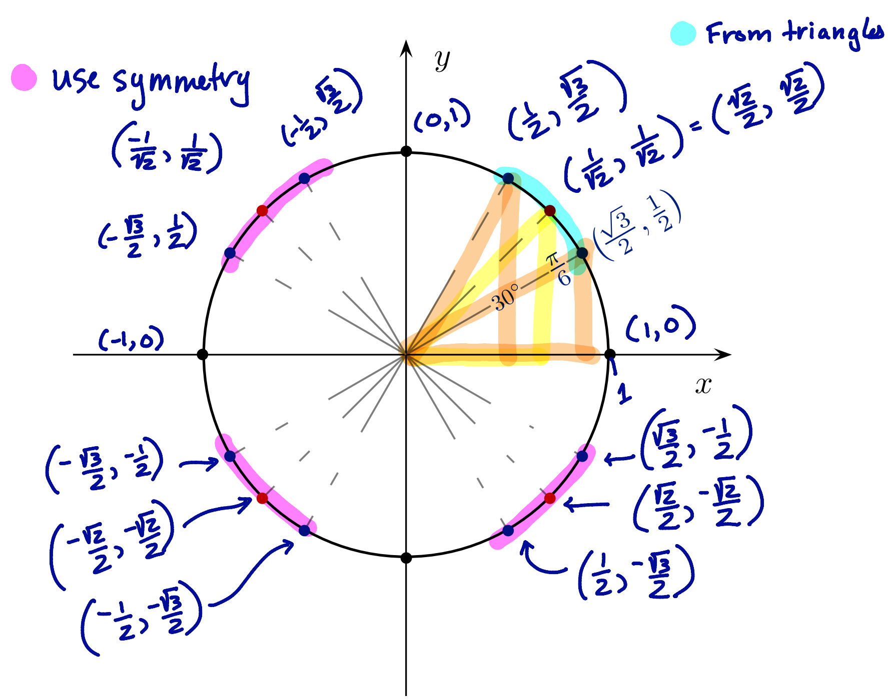
hypotenuse: $\sqrt{10^2+4^2} = \sqrt{116} = 2\sqrt{29}$

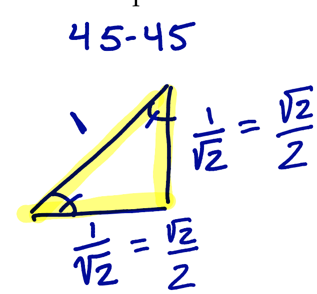
$\sin\theta = \frac{10}{2\sqrt{29}} = \frac{5}{\sqrt{29}}$

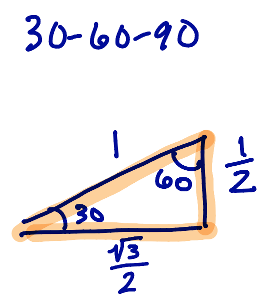
$\cos\theta = \frac{2}{\sqrt{29}}$

The Unit Circle Approach

5. Using a 45-45-90 triangle and a 30-60-90 triangle find the coordinates of ALL of the points on the unit circle.

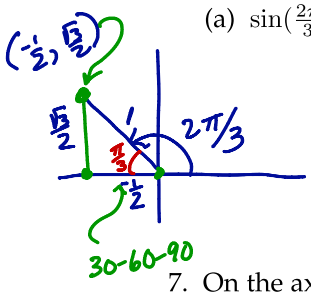






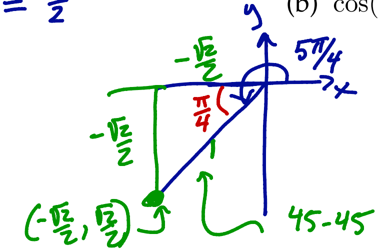
6. Without a calculator evaluate:

(a) $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$

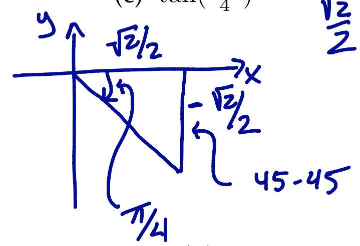


(b) $\cos(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2}$

$\frac{5\pi}{4} = \pi + \frac{1}{4}\pi$

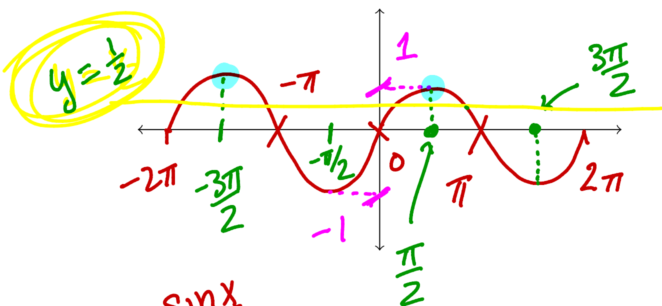


(c) $\tan(\frac{-\pi}{4}) = \frac{-\sqrt{2}/2}{\frac{\sqrt{2}}{2}} = -1$

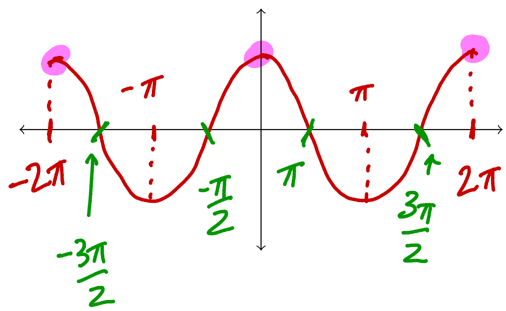


7. On the axes below, graph *at least two cycles* of $f(x) = \sin x$, $f(x) = \cos x$, and $f(x) = \tan x$. Label all x- and y-intercepts.

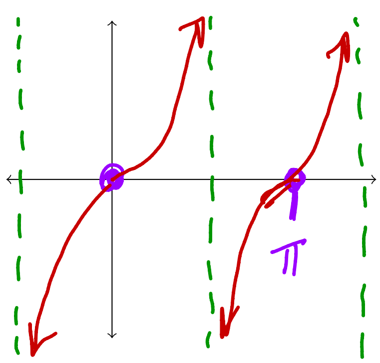
$y = \sin x$



$y = \cos x$



$y = \tan x = \frac{\sin x}{\cos x}$



← where $\sin x = 0$

$x = -\frac{\pi}{2}$ $\quad x = \frac{\pi}{2}$ $\quad x = \frac{3\pi}{2}$ ← where $\cos x = 0$

8. Use the graphs above to solve the equations below.

(a) $\cos x = 1$

$x = \ldots, -2\pi, 0, 2\pi, 4\pi, \ldots$

$= 2\pi k$, k integer

(b) $\sin x = 1$

$x = \ldots \frac{\pi}{2} - 2\pi, \frac{\pi}{2}, \frac{\pi}{2} + 2\pi, \frac{\pi}{2} + 4\pi, \ldots$

$= \frac{\pi}{2} + 2\pi k$, k integer

$= \ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

(c) $\tan x = 0$

$x = \ldots, -\pi, 0, \pi, 2\pi, \ldots$

$= \pi k$, k integer

(d) $\sin x = 1/2$

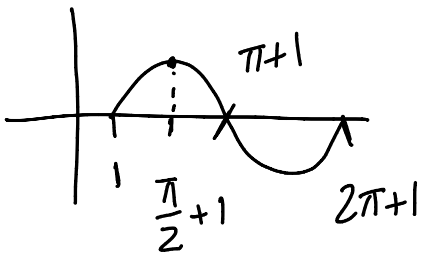
think: $\theta = \frac{\pi}{6}$

- (Find all solutions in $[0, 2\pi]$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

9. For each problem below, sketch the graph and use it to help you solve the equation or answer the question.

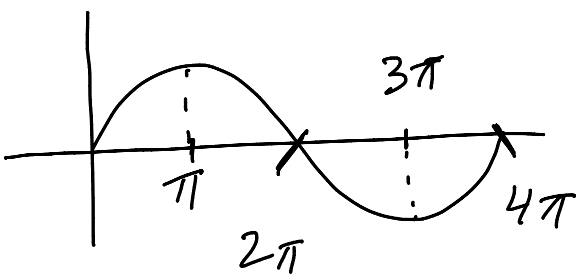
(a) Graph $y = \sin(x-1)$ and use it to solve the equation $\sin(x-1) = 1$.



$\sin(x-1) = 1$ requires

$x = \frac{\pi}{2} + 1 + 2\pi k$, k integer

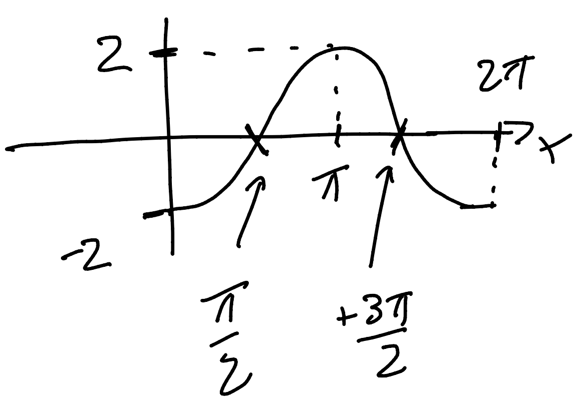
(b) Graph $y = \sin(x/2)$ and use it to find the domain of $f(x) = \csc(x/2)$.



$\csc(x/2) = \frac{1}{\sin(x/2)}$ avoid 0 in denominator

domain:
all real numbers except πk,
k integer

(c) Graph $y = -2\cos(x)$ and use it to solve the equation $-2\cos(x) = 0$.



$x = \ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$= \frac{\pi}{2} + \pi k$, k integer