Lecture Notes: Review of Chapters 3 & 4

Summary of Topics

Chapter 3

- Recall Sections 1-6 involve derivative rules. This will *not* be explicitly tested.
- Sections 7 and 8 focus on applications of the derivative in science and particularly to exponential growth and decay. Position, velocity and acceleration were again discussed. The overall emphasis is on *interpretation* of the derivative in the context of an applied problem.
- Section 9 Related Rate Problems. In these problems you are always taking the derivative implicitly with respect to time and almost always seeking of find a rate of change at a particular instant.
- Section 10 Linear Approximations and Differentials. The crucial idea here is that the derivative can be used to estimate function-values or changes in function-values.
- Section11 we did not cover.

Chapter 4

- Section 1 make a careful study of the ideas of local/absolute maximum/minimum and the difference between an extreme value and where it occurs.
- Section 4.2 The Mean Value Theorem. Know, roughly, what it says and be able to draw a picture.
- Section 4.3 discussed how the sign of f' and f'' can tell us things about f such as intervals on which f is increasing, decreasing, concave up, concave down, local/absolute extreme values.
- Section 4.4 involved L'Hôpital's Rule. Recall that before using this rule one should make sure it applies.
- Section 4.5 put a whole bunch of Calculus together to sketch a graph. In addition to topics from Section 1 and 2, we also included things like x- and y-intercepts, vertical and horizontal asymptotes, and the function's domain.
- Section 4.6 was not discussed.
- Section 4.7 involved Optimization. Recall that by this time we have a clear understanding of how the domain of the function may determine the techniques we use to determine the answer.
- Section 4.8 will be discussed at the end of the semester and will not appear on this midterm.
- Section 4.9 involves antiderivatives.

1. Find the domain of the function $f(x) = \frac{\sin(5x)}{x^2+x}$ and identify any vertical or horizontal asymptotes. Justify your answer.

$f(x) = \dfrac{\sin(5x)}{x(x+1)}$ domain: $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

H.A.: $\lim\limits_{x\to\infty} \dfrac{\sin(5x)}{x(x+1)} = 0$. So $y=0$

V.A. $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{x^2+x} \overset{H}{=} \lim\limits_{x\to 0} \dfrac{5\cos(5x)}{2x+1} = \dfrac{5}{1} \neq \pm\infty$. So $x=0$ NOT asymptote.

2. $f(x) = (x-4)\sqrt[3]{x} = x^{4/3} - 4x^{1/3}$; $f'(x) = \frac{4(x-1)}{3x^{2/3}}$; $f''(x) = \frac{4(x+2)}{9x^{5/3}}$.

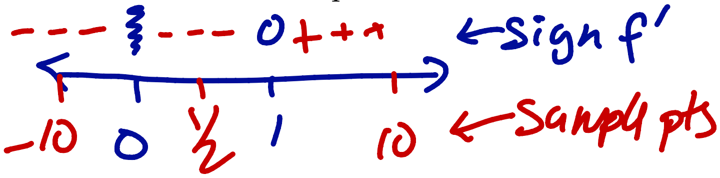
(a) Find the critical numbers of $f(x)$.

$f' = 0$ when $x = 1$

f' undef. when $x = 0$

$\boxed{\text{answer: } x=0,\ x=1}$

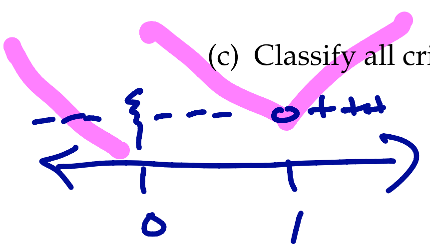
(b) Find the open intervals on which the function is increasing or decreasing.



f increasing on $(1,\infty)$

decreasing on $(-\infty,0)\cup(0,1)$

(c) Classify all critical points – using the **first derivative test**.



local min at $x=1$

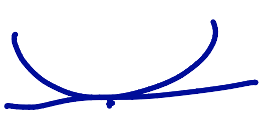
no local max

$x=0$ is neither a l. min nor l. max

(d) Classify all critical points – using the **second derivative test**.

$f''(0)$ undefined. (Test fails ☹)

$f''(1) > 0$ so l. min at $x=1$.



$$\lim_{x\to -1} \frac{\sin(5x)}{x^2+x} = \pm\infty$$ since $\sin(5x) \to \sin(-5)$ and $x^2+x \to 0^{+/-}$

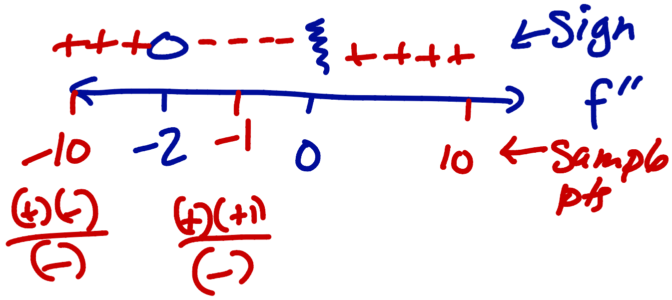
So $x=-1$ is a vertical asymptote.

2. $f(x) = (x-4)\sqrt[3]{x} = x^{4/3} - 4x^{1/3}$; $f'(x) = \frac{4(x-1)}{3x^{2/3}}$; $f''(x) = \frac{4(x+2)}{9x^{5/3}}$.

(e) Find the open intervals on which the function is concave up or concave down.

$f''=0$ when $x=-2$

f'' undf. when $x=0$



f is cup on $(-\infty,-2)\cup(0,\infty)$

and ccdown on $(-2,0)$

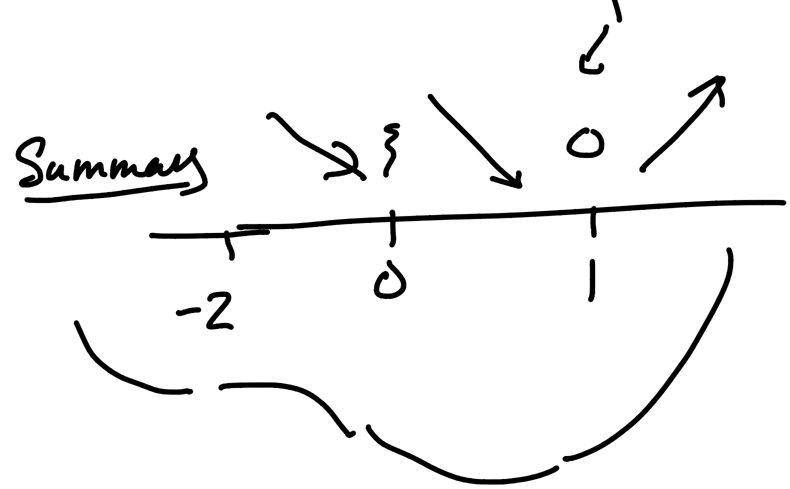
(f) Find the inflection points.

Inflection points at $(-2, 6\sqrt[3]{2})$ and $(0, \underline{0})$

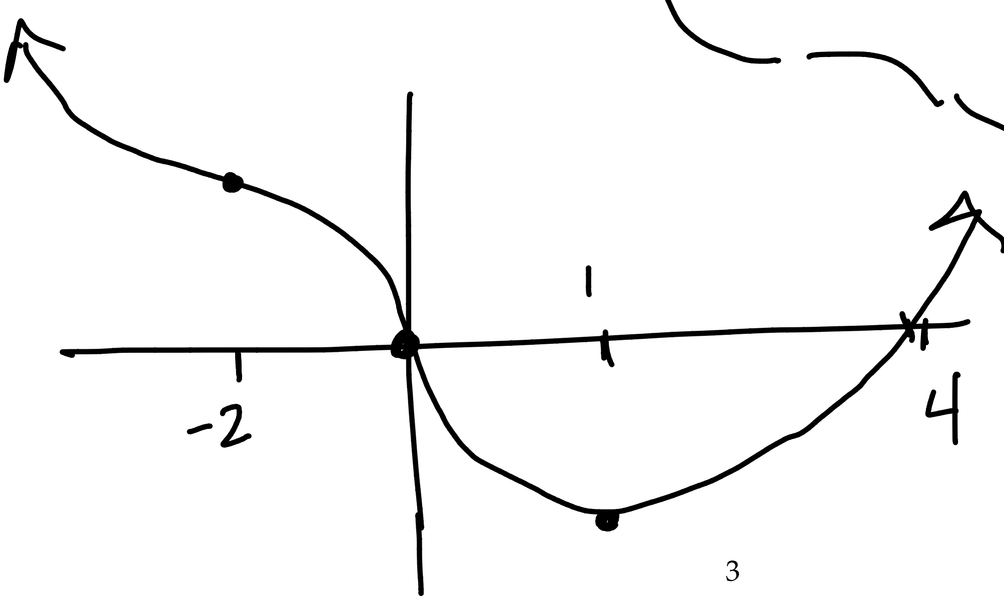
$f(-2) = -6(-2)^{1/3}$
$= 6\sqrt[3]{2}$

$f(0) = 0$

$f(1) = -3$



(g) Sketch the graph.



3. Find the linearization of $f(x) = \sqrt{x}$ at $a = 4$ and use it to estimate $\sqrt{4.1}$ and 3.8.

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f(4) = 2$

$f'(4) = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x-4)$

linearization $L(x) = 2 + \frac{1}{4}(x-4)$

$\sqrt{4.1} \approx 2 + \frac{1}{4}(4.1-4) = 2 + \frac{1}{40} = \frac{81}{40}$

$\sqrt{3.8} \approx 2 + \frac{1}{4}(3.8-4) = 2 - \frac{1}{20} = \frac{39}{40}$

4. Find the differential of $y = \sqrt{x}$ and use it to estimate how much y will change as x changes from $x = 4$ to $x = 4.1$.

$y = x^{1/2}$

$\boxed{dy = \frac{1}{2}x^{-1/2}dx}$

estimate $\Delta y \approx \frac{1}{2}(4)^{-1/2}\left(\frac{1}{10}\right)$

$= \frac{1}{40}$

5. Evaluate the following limits. Show your work.

(a) $\lim_{x\to 0} \frac{1 + x - e^x}{\sin x}$ $\leftarrow \frac{0}{0}$

$\overset{H}{=} \lim_{x\to 0} \frac{1-e^x}{\cos x} = \frac{0}{1} = 0$

(b) $\lim_{x\to\infty} x \ln(1 + \frac{2}{x})$ $\leftarrow$ form $\infty \cdot 0$

$= \lim_{x\to\infty} \frac{\ln(1+2x^{-1})}{x^{-1}}$ $\leftarrow$ form $\frac{0}{0}$

$\overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{1+2x^{-1}} \cdot -2x^{-2}}{-x^{-2}} =$

$= \lim_{x\to\infty} \frac{2}{1+2x^{-1}} = 2$