LECTURE NOTES: REVIEW OF CHAPTERS 3 & 4

Summary of Topics

Chapter 3

- Recall Sections 1-6 involve derivative rules. This will *not* be explicitly tested.
- Sections 7 and 8 focus on applications of the derivative in science and particularly to exponential growth and decay. Position, velocity and acceleration were again discussed. The overall emphasis is on *interpretation* of the derivative in the context of an applied problem.
- Section 9 Related Rate Problems. In these problems you are always taking the derivative implicitly with respect to time and almost always seeking of find a rate of change at a particular instant.
- Section 10 Linear Approximations and Differentials. The crucial idea here is that the derivative can be used to estimate function-values or changes in function-values.
- Section11 we did not cover.

Chapter 4

- Section 1 make a careful study of the ideas of local/absolute maximum/minimum and the difference between an extreme value and where it occurs.
- Section 4.2 The Mean Value Theorem. Know, roughly, what it says and be able to draw a picture.
- Section 4.3 discussed how the sign of f' and f'' can tell us things about f such as intervals on which f is increasing, decreasing, concave up, concave down, local/absolute extreme values.
- Section 4.4 involved L'Hôpital's Rule. Recall that before using this rule one should make sure it applies.
- Section 4.5 put a whole bunch of Calculus together to sketch a graph. In addition to topics from Section 1 and 2, we also included things like x- and y-intercepts, vertical and horizontal asymptotes, and the function's domain.
- Section 4.6 was not discussed.
- Section 4.7 involved Optimization. Recall that by this time we have a clear understanding of how the domain of the function may determine the techniques we use to determine the answer.
- Section 4.8 will be discussed at the end of the semester and will not appear on this midterm.
- Section 4.9 involves antiderivatives.

1. Find the linearization of $f(x) = \sqrt{x}$ at $a = 4$ and use it to estimate $\sqrt{4.1}$ and $\sqrt{3.8}$.

2. Find the differential of $y = \sqrt{x}$ and use it to estimate how much y will change as x changes from $x = 4$ to $x = 4.1$.

3. Evaluate the following limits. Show your work.

   (a) $\lim_{x \to 0} \dfrac{1 + x - e^x}{\sin x}$

   (b) $\lim_{x \to \infty} x \ln(1 + \dfrac{2}{x})$

4. Find the domain of the function $f(x) = \frac{\sin(5x)}{x^2+x}$ and identify any vertical or horizontal asymptotes. Justify your answers.

5. $f(x) = (x-4)\sqrt[3]{x} = x^{4/3} - 4x^{1/3}$; $\quad f'(x) = \frac{4(x-1)}{3x^{2/3}}$; $\quad f''(x) = \frac{4(x+2)}{9x^{5/3}}$.

   (a) Find the critical numbers of $f(x)$.

   (b) Find the open intervals on which the function is increasing or decreasing.

   (c) Classify all critical points – using the **first derivative test**.

From the other side: $f(x) = (x-4)\sqrt[3]{x} = x^{4/3} - 4x^{1/3}$; $\quad f'(x) = \frac{4(x-1)}{3x^{2/3}}$; $\quad f''(x) = \frac{4(x+2)}{9x^{5/3}}$.

(d) Classify all critical points – using the **second derivative test**.

(e) Find the open intervals on which the function is concave up or concave down.

(f) Find the inflection points.

(g) Sketch the graph.