Summary of Topics

Our final exam will be TUESDAY December 10 from 1:00pm-3:00pm. Jill Faudree's class will be in Chapman 106. Leah Berman's class will be in Duck 342. The Final Exam will be cumulative. You will have 2 hours to complete it. Books, notes, calculators and other aids are not allowed.
As with all assessments in this course, you are strongly encouraged to work some old Final Exams as practice.

Sample Problems

1. Given $f(x) = 3x - x^2$, find $f'(x)$ *using the definition of the derivative.*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)-(x+h)^2-(3x-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x}+3h-\cancel{x^2}-2xh-h^2-\cancel{3x}+\cancel{x^2}}{h} = \lim_{h\to 0} \frac{3h-2xh-h^2}{h}$$

$$= \lim_{h\to 0} 3-2x-h = 3-2x$$

2. Find the equation of the line tangent to $ye^x + 2 = x^2 + y^2$ at the point $(0, 2)$.

Find $\frac{dy}{dx}$: $\frac{dy}{dx}\cdot e^x + ye^x = 2x + 2y\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2x-ye^x}{e^x-2y}$$

$$\left.\frac{dy}{dx}\right|_{(0,2)} = \frac{2\cdot 0-2\cdot e^0}{e^0-2\cdot 2} = \frac{-2}{-3} = +\frac{2}{3} = m$$

$$y-2 = +\frac{2}{3}(x-0)$$

$$\boxed{y = 2+\frac{2}{3}x}$$

3. Let $F(t) = \frac{20}{4+e^{-2t}}$ model the population of fish in hundreds over time t measured in years.

(a) Find and interpret $F(0)$.

$F(0) = \frac{20}{5} = 4$ hundred fish.

At the start of the measurements, there were 400 fish.

(b) Find and interpret (in language your parents could understand) $\lim_{t\to\infty} F(t)$.

$\lim_{t\to\infty} \frac{20}{4+e^{-2t}} = \frac{20}{4} = 5$, Long term, the population stabilizes at 500 fish.

(c) Find $F'(t)$. (HINT: You can check your answer with the one at the bottom of the page.

$F(t) = 20(4+e^{-2t})^{-1}$

$F'(t) = 20(-1)(4+e^{-2t})^{-2}(-2e^{-2t}) = \frac{40}{e^{2t}(4+e^{-2t})^2}$

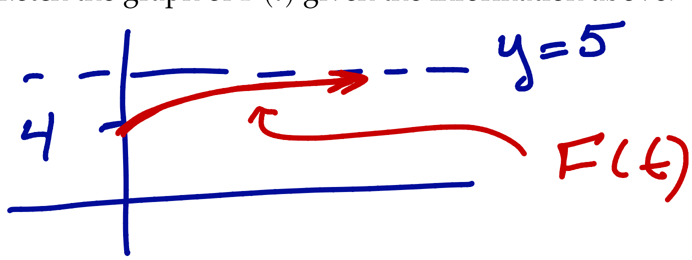
(d) Find and interpret $F'(0)$.

$F'(0) = \frac{40}{1(4)^2} = \frac{40}{16} = \frac{5}{2}$ (in hundreds of fish per year)

(e) Find and interpret (in language your parents could understand) $\lim_{t\to\infty} F'(t)$.

$\lim_{t\to\infty} \frac{40}{e^{2t}(4+e^{-2t})^2} = 0$. Long term, the rate at which the population increases goes to zero. That is, the population stops changing.

(f) Give a rough sketch the graph of $F(t)$ given the information above.



$F'(t) = \frac{40e^{-2t}}{(4+e^{-2t})^2}$

4. Let $f(x) = \frac{5x^2}{1-\cos(x)}$.

(a) Find $\lim_{x\to 0} \frac{5x^2}{1-\cos x}$ $\overset{H}{=} \lim_{x\to 0} \frac{10x}{+\sin x} \overset{H}{=} \lim_{x\to 0} \frac{10}{\cos x} = 10$

form $\frac{0}{0}$ form $\frac{0}{0}$

(b) Does $f(x)$ have a vertical asymptote at $x = 0$? Explain

No. $\lim_{x\to 0} f(x) \neq \pm\infty$. So, no vertical asymptote.

5. Let $g(x) = \frac{4x^4+5}{(x^2-2)(2x^2-1)}$. Does $g(x)$ have any horizontal asymptotes? Justify your answer with a limit.

$\lim_{x\to\infty} \frac{4x^4+5}{(x^2-2)(2x^2-1)} = \frac{4}{2} = 2$. So $g(x)$ has $y=2$ as a horizontal asymptote.

6. Complete two iterations of Newton's Method to estimate a solution to $x^7 + 4 = 0$. Use $x_0 = -1$.

$f(x) = x^7+4$

$f'(x) = 7x^6$

$x_{k+1} = x_k - \frac{x_k^7+4}{7x_k^6}$

$x_0 = -1$

$x_1 = x_0 - \frac{x_0^7+4}{7x_0^6} = -1 - \frac{-1+4}{7} = -1 - \frac{3}{7} = \frac{-10}{7}$

$x_2 = x_1 - \frac{x_1^7+4}{7x_1^6} = \frac{-10}{7} - \frac{\left(\frac{-10}{7}\right)^7+4}{7\left(\frac{10}{7}\right)^6}$

7. Evaluate.

(a) $\int_0^{\pi/4} \frac{\sec^2 t}{\tan(t)+1} dt$ $= \ln|\tan t + 1| \Big]_0^{\frac{\pi}{4}} = \ln(\tan\frac{\pi}{4}+1) - \ln(\tan 0 + 1)$

$= \ln(2) - \ln(1) = \ln(2)$

(b) $\int_0^8 \frac{3}{\sqrt{x+1}} dx$ $= \int_0^8 3(x+1)^{-1/2} dx = 6(x+1)^{1/2}\Big]_0^8 = 6\left[9^{1/2} - 1^{1/2}\right] = 6(3-1) = 12$

8. A particle is moving with velocity $v(t) = 2t - \frac{1}{1+t^2}$ measured in meters per second.

(a) Find and interpret $v(0)$.

$V(0) = 0 - 1 = -1$ m/s. The particle is moving backwards at this time.

(b) Find the displacement for the particle from time $t = 0$ to time $t = 4$. Give units with your answer.

$\int_0^4 \left(2t - \frac{1}{1+t^2}\right) dt = t^2 - \arctan t\Big]_0^4 = 16 - \arctan(4)$ meters

(c) If D is the *distance* the particle traveled over the interval $[0, 4]$, is D larger or smaller or exactly the same as your answer in part (b)? Justify your answer.

D will be larger. At time $t=4$, $V(4) = 16 - \frac{1}{17} > 0$. So by the end of the interval, the particle is moving forward. Since the particle changes directions, distance will be larger than displacement.

(d) Assuming $s(0) = 1$, find the position of the particle. at any time t

$s(t) = t^2 - \arctan t + C$

$s(0) = 0 - 0 + C = 1$

So $C = 1$.

$s(t) = t^2 - \arctan t + 1$