§2.7 Starter Notes

Recall §2.1 #3

$P(2,-1)$ lies on $y = \frac{-1}{x-1}$

Goal: Find SLOPE of secant line PQ where Q takes x-values below:

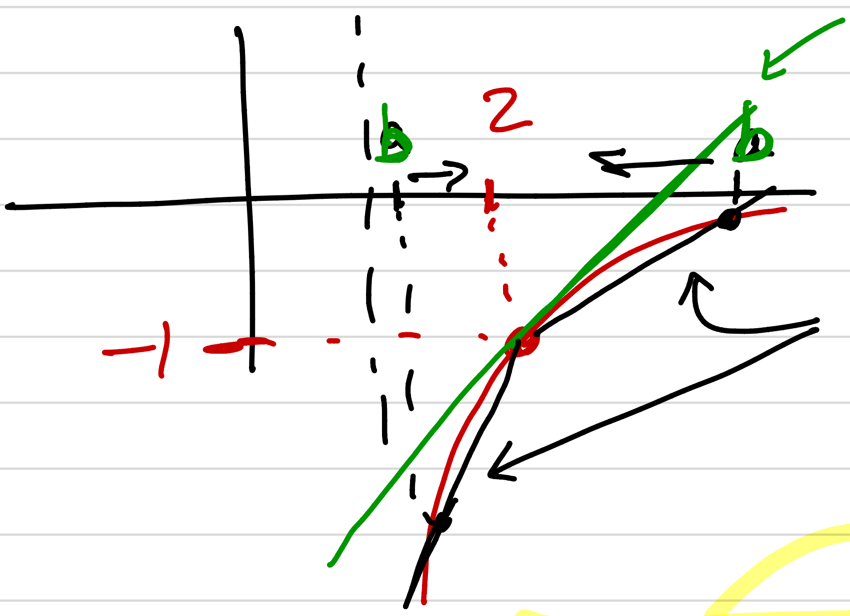
x	1.5	1.9	1.99	1.999	2.001	2.01	2.1	2.5
m_{sec}	2	1.1111	1.010101	1.001001..	0.9990999..	0.990990..	0.909090..	0.6666

How did you do this calculation? $x = \frac{3}{2}$, $y = \frac{-1}{\frac{3}{2}-1} = \frac{-1}{\frac{1}{2}} = -2$

$$m = \frac{\Delta y}{\Delta x} = \frac{-1-(-2)}{2-\frac{3}{2}} = \frac{1}{\frac{1}{2}} = 2$$

Conclusion: $\boxed{\text{slope of tangent to } y = \frac{-1}{x-1} \text{ at } x = \boxed{2}} = \boxed{1}$ follows from *

Picture:



slope is = 1

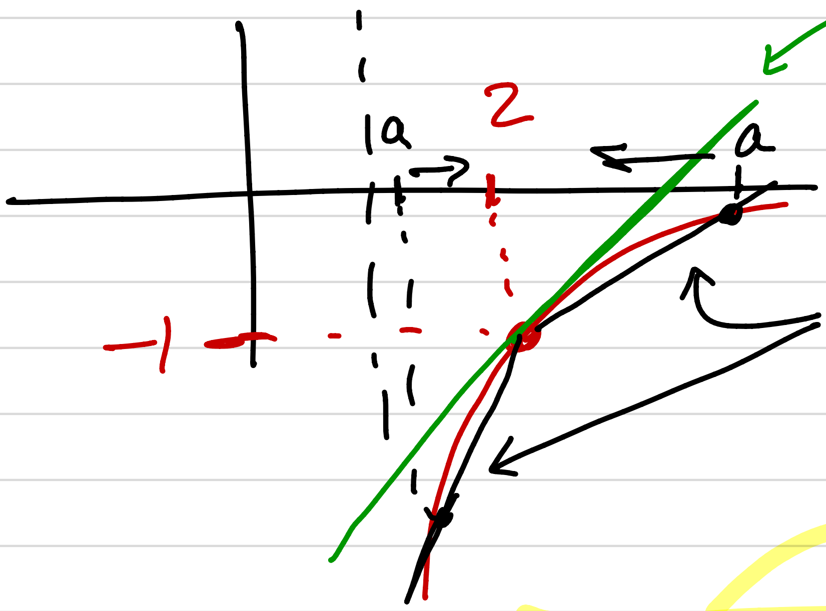
secant lines where a gets close to 2.

slope of secant

algebra: $m_{tan} = \lim_{b \to 2} \boxed{\frac{f(b) - f(2)}{b-2}} = \underbrace{f'(2)}_{\text{called the derivative of } f(x) \text{ at } x=2.}$

From the other side:

Picture:



slope is = 2

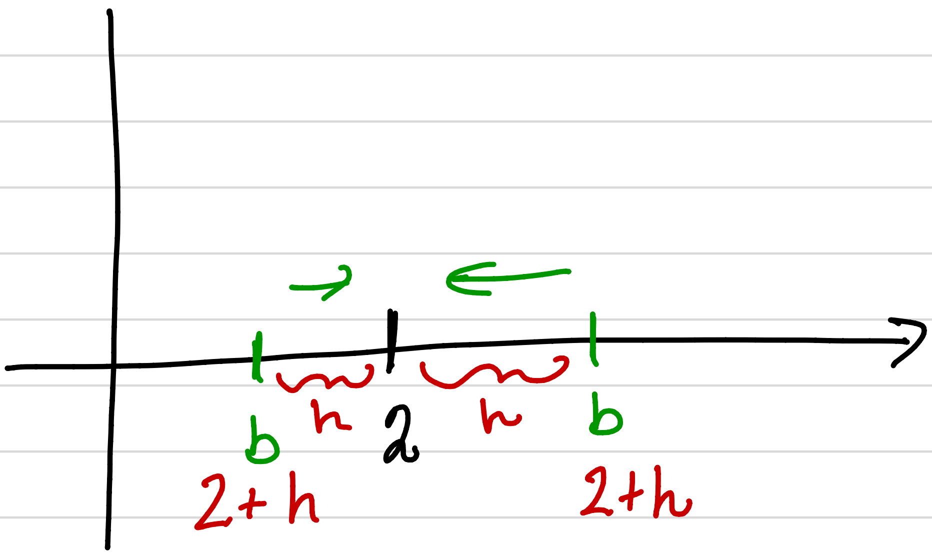
Secant lines where a gets close to 2.

Slope of secant

algebra:

$$m_{tan} = \lim_{b \to 2} \boxed{\frac{f(b) - f(2)}{b - 2}} = \underbrace{f'(2)}_{\text{called the derivative of } f(x) \text{ at } x=2.}$$

A change of notation makes algebra easier:



$b \to 2$

replaced by

$h \to 0$

(So $\boxed{b = 2+h} \to \boxed{2}$)

$$\text{So} \quad \lim_{b \to 2} \frac{f(b) - f(2)}{b - 2} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{2+h-2} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

Recall, we know $f'(2)=1$ from previous work!

A Specific Example

Find $f'(2)$ for $f(x) = \dfrac{-1}{x-1}$.

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{\frac{-1}{2+h-1} - \left(\frac{-1}{1}\right)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{-1}{1+h} + 1\right) = \lim_{h \to 0} \frac{1}{h}\left(\frac{-1+1+h}{1+h}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{h}{1+h}\right) = \lim_{h \to 0} \left(\frac{1}{1+h}\right) = 1 \quad \checkmark$$

Big Summary:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$