Example 3: Find derivatives of the following functions.

(a) $f(x) = \log_{10}\sqrt{x}$

$$f'(x) = \frac{1}{\ln 10\sqrt{x}} \cdot \frac{d}{dx}\sqrt{x}$$
$$= \frac{1}{\ln 10\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$$
$$= \frac{1}{\ln 10\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2\ln 10 x}}$$

(b) $g(x) = \log_2(\cos x)$

$$g'(x) = \frac{1}{\ln 2 \cos x} \cdot (-\sin x)$$
$$= \boxed{-\frac{\tan x}{\ln 2}}$$

Example 4: Differentiate f and find the domain of f'.

(a) $f(x) = \sqrt{5 + \ln x} = (5+\ln x)^{1/2}$

$$f'(x) = \frac{1}{2}(5+\ln x)^{-1/2} \cdot \frac{d}{dx}(5+\ln x)$$
$$= \frac{1}{2\sqrt{5+\ln x}} \cdot \frac{1}{x}$$
$$= \boxed{\frac{1}{2x\sqrt{5+\ln x}}}$$

need $x > 0$ for $\ln x$ and $5+\ln x \geq 0$

$\ln x \geq -5$

$\boxed{x \geq e^{-5}}$

(b) $f(x) = \dfrac{x}{1 - \ln(x+1)}$

$$f'(x) = \left(\frac{1-\ln(x+1) - x\cdot\left(\frac{-1}{x+1}\right)}{(1-\ln(x+1))^2}\right)\frac{\frac{x+1}{1}}{\frac{x+1}{1}}$$
$$= \frac{x+1-(x+1)\ln(x+1)+x}{(x+1)(1-\ln(x+1))^2}$$
$$= \boxed{\frac{2x+1-x\ln(x+1)-\ln(x+1)}{(x+1)(1-\ln(x+1))^2}}$$

$x \neq -1 \quad x+1 > 0 \Rightarrow x > -1$

also $1 - \ln(x+1) \neq 0$

$1 \neq \ln(x+1)$

$e \neq x+1$, so $x \neq e-1$

$\boxed{D: (-1, e-1) \cup (e-1, \infty)}$

Example 5: Differentiate the following functions.

(a) $y = \ln|x|$.

case 1

if $x > 0$ (positive) then $|x| = x$

and $y = \ln|x| = \ln x$; $y' = 1/x$

case 2

if $x < 0$ (neg) then $|x| = -x$

and $y = \ln|x| = \ln(-x)$; $y' = \frac{1}{-x}(-1) = 1/x$

so if $y = \ln|x|$;

$\boxed{y' = 1/x \text{ w/no abs!}}$

(b) $f(x) = \ln|\sec x + \tan x|$

$$f'(x) = \frac{1}{\sec x + \tan x} \cdot \frac{d}{dx}(\sec x + \tan x)$$
$$= \frac{\sec x\tan x + \sec^2 x}{\sec x + \tan x}$$
$$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$
$$= \boxed{\sec x}$$

It is often easier to first use the rules of logarithms to expand a logarithmic expression before taking the derivative. To do this properly you first must recognize when these rules can be applied and apply them correctly.

Rules and Non-Rules for Logarithms

- $\ln(AB) =$ $\ln A + \ln B$
- $\ln(A/B) =$ $\ln A - \ln B$
- $\ln(A^r) =$ $r \ln A$
- $\ln(A + B) =$ No rule $\neq \ln A + \ln B$
- $\ln(A - B) =$ No rule $\neq \ln A - \ln B$
- $(\ln A)^r =$ No rule $\neq r \ln A$

Example 6: Differentiate the following functions by first expanding the expressions using the rules for logarithms. Explain *why* this is the better way to proceed in each case.

(a) $f(x) = \ln \sqrt{5x + 2}$

$= \ln(5x+2)^{1/2}$

$= \frac{1}{2}\ln(5x+2)$

$f'(x) = \frac{1}{2} \cdot \left(\frac{1}{5x+2}\right) \cdot 5$

$= \boxed{\frac{5}{2(5x+2)}}$

would have had to chain twice w/out expansion

(b) $g(x) = \log_5(x^2\sqrt{x + 1})$

$= 2\log_5 x + \frac{1}{2}\log_5(x+1)$

$g'(x) = \frac{2}{(\ln 5)x} + \frac{1}{2}\frac{1}{(\ln 5)(x+1)}$

$= \frac{2}{(\ln 5)x}\frac{2(x+1)}{2(x+1)} + \frac{1}{2}\frac{1}{(\ln 5)(x+1)}\frac{x}{x}$

$= \frac{4x+4+x}{2\ln 5\, x(x+1)}$

$= \boxed{\frac{5x+4}{2\ln 5\, x(x+1)}}$

Example 7: Differentiate $f(x) = \ln\left(\frac{x(x^2+1)^2}{\sqrt{2x^4-5}}\right)$

$f(x) = \ln x + 2\ln(x^2+1) - \frac{1}{2}\ln(2x^4-5)$

$f'(x) = \frac{1}{x} + \frac{2}{x^2+1} \cdot 2x - \frac{1}{2}\left(\frac{1}{2x^4-5}\right) \cdot 8x^3$

$= \boxed{\frac{1}{x} + \frac{4x}{x^2+1} - \frac{4x^3}{2x^4-5}}$