Reminders about Inverse Functions and $f(x) = \log_b x$.

Inverse functions: $f(x)$, $f^{-1}(x)$ ← inverse NOT $\frac{1}{f(x)} = (f(x))^{-1}$

Switch input & output

Switch x and y

$a \rightarrow \boxed{f(x)} \rightarrow b$ $\qquad$ $b \rightarrow \boxed{f^{-1}(x)} \rightarrow a$

f^{-1} "un does" f

If $f(x) = x + 10$, then $f^{-1}(x) = x - 10$

What is the inverse of $f(x) = e^x$? $\qquad f^{-1}(x) = \ln x$

$y = e^x$ $\qquad$ $y = \ln x$ OR $x = e^y$

Switch x and y

Numerical Example: $\log_{10} \frac{1}{1000} = y$ OR $10^y = \frac{1}{1000} = 10^{-3}$

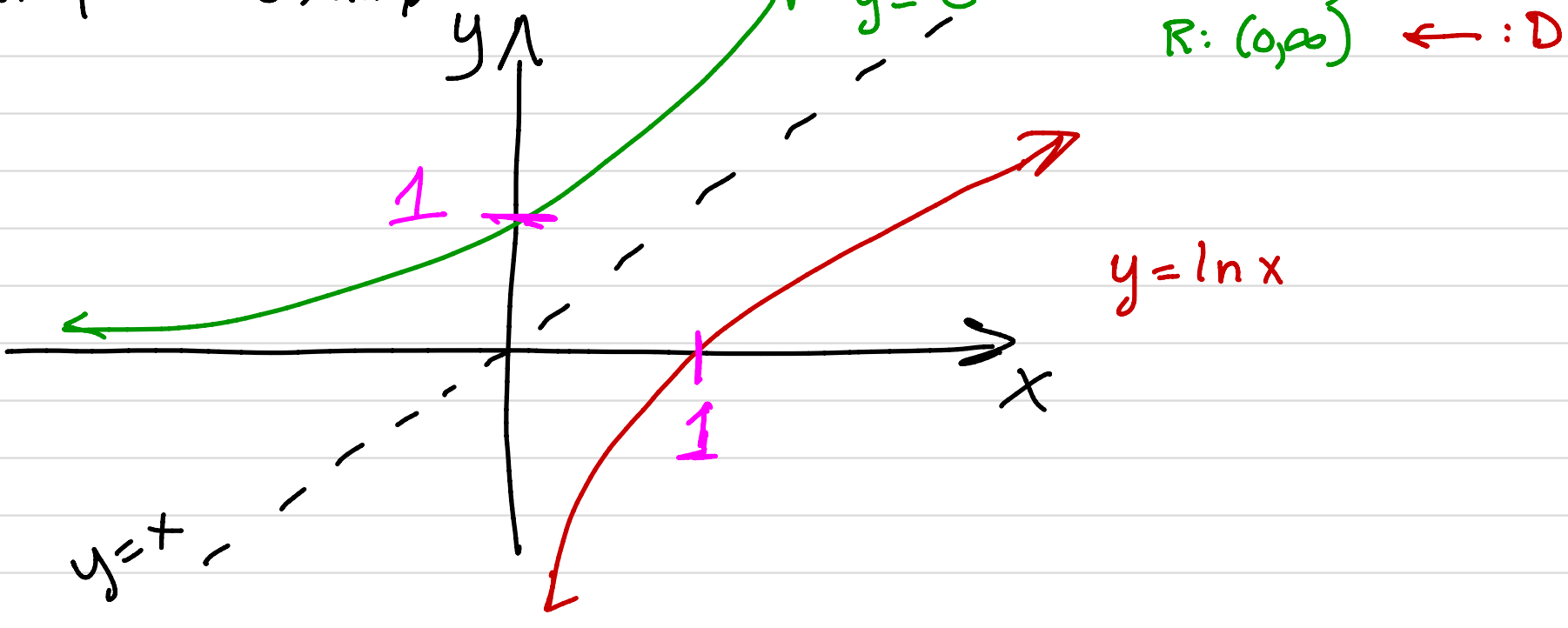
So $y = -3$

Algebraic Example: Solve $e^x = 20$.

$\ln e^x = \ln 20$

$x = \ln 20$

Graphical Example:



D: $(-\infty, \infty)$ ← : R

R: $(0, \infty)$ ← : D

LECTURE NOTES: §1.5

1. Without doing a bunch of algebra, find $f^{-1}(x)$ for each function below:

   (a) $f(x) = 2x$

   $f^{-1}(x) = \frac{1}{2}x$

   (b) $f(x) = x^3$

   $f^{-1}(x) = \sqrt[3]{x}$

2. Without explicitly finding a formula for $f^{-1}(x)$, find $f^{-1}(1)$ for each function below:

   (a) $f(x) = x - 20$

   Since $f(21) = 21 - 20 = 1$,

   $f^{-1}(1) = 21$

   (b)

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2.0
$f(x)$	20	10	5	3	2.5	2	1.5	1	0.25

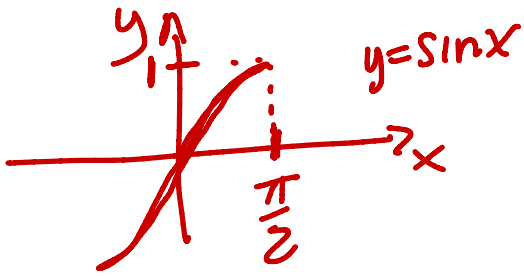
   Since $f(1.75) = 1$,

   $f^{-1}(1) = 1.75$

3. Evaluate $\sin^{-1}(1)$.

   Since $\sin\left(\frac{\pi}{2}\right) = 1$,

   $\sin^{-1}(1) = \frac{\pi}{2}$.



4. Find the exact value of each expression.

   (a) $\log_2 16$

   $\log_2 2^4 = 4$

   (b) $e^{\ln 5} = 5$

5. Solve each equation below for x.

(a) $10 = 2e^{x+1}$

$5 = e^{x+1}$

$\ln 5 = x+1$

$x = (\ln 5) - 1$

(b) $\ln(x^2 - 1) = 1$

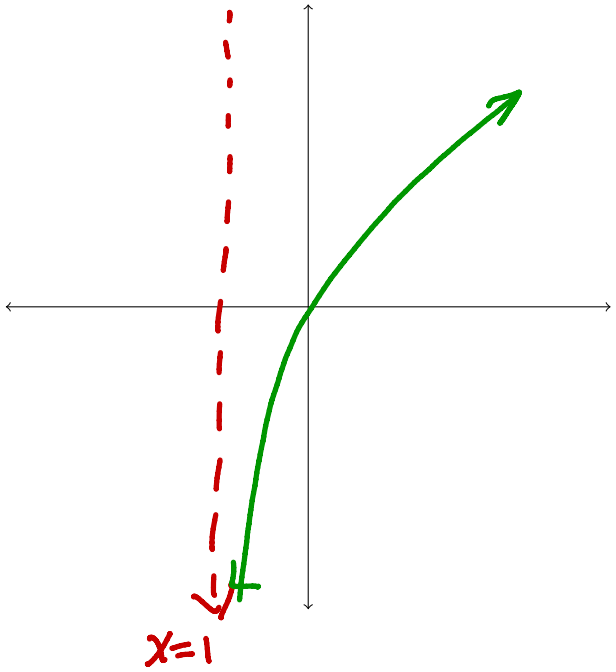
$x^2 - 1 = e^1$

$x^2 = e + 1$

$x = \pm\sqrt{e+1}$

6. Sketch each function. Include domain, range, intercepts and asymptotes.

(a) $f(x) = \ln(x+1)$



(b) $f(x) = -\ln x$ reflect about x-axis