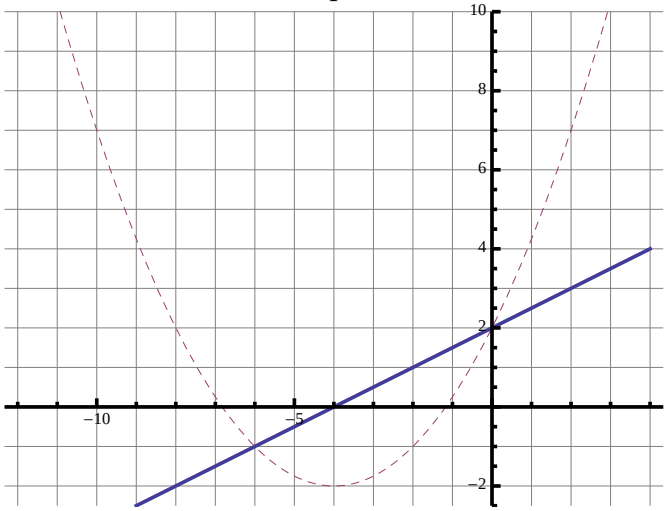
WORKSHEET §2.8: FINDING DERIVATIVES GRAPHICALLY - SOLUTIONS

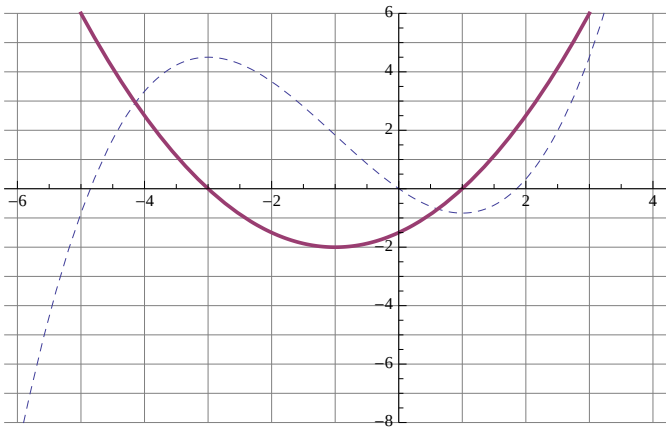
The original functions are shown dashed, and the derivatives are shown thick.

Exercise 1. Sketch the derivatives of graphs 1 and 2.

Graph 1



Graph 2



Exercise 2. The equation of Graph 1 is

$$f(x) = 2x + \frac{x^2}{4}.$$

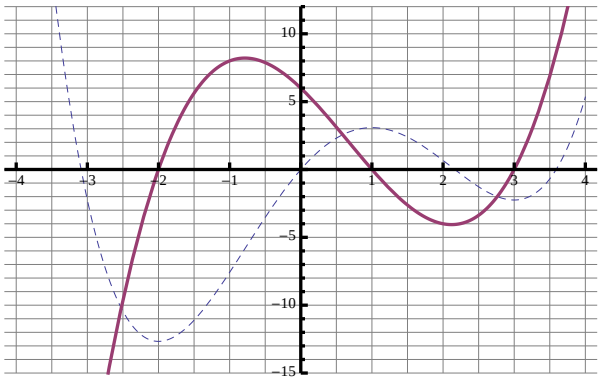
Use the definition of the derivative to compute the derivative $f'(x)$. (Attach a separate page if you need more room.) What kind of function is $f'(x)$? How does the graph of $f'(x)$ compare to the derivative you drew?

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\left(2(x+h) + \frac{(x+h)^2}{4}\right) - \left(2x + \frac{x^2}{4}\right)}{h} \\ &= \lim_{h\to 0} \frac{1}{h}\left(\cancel{2x} + 2h + \frac{1}{\cancel{4}}\cancel{x^2} + \frac{1}{4}\cdot 2xh + \frac{1}{4}(h)^2 - \cancel{2x} - \frac{1}{\cancel{4}}\cancel{x^2}\right) \\ &= \lim_{h\to 0} \frac{1}{\cancel{h}}\cdot \cancel{h}\left(2 + \frac{1}{4}\cdot 2x + \frac{1}{4}(h)\right) \\ &= 2 + \frac{x}{2} \end{aligned}$$

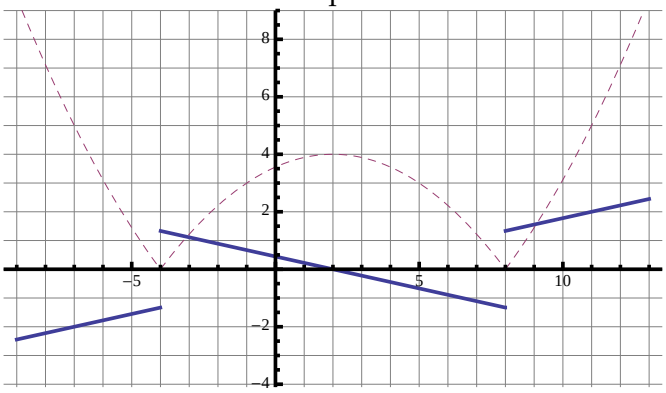
Look! The derivative is a line! And hopefully you sketched something line-like for Graph 1.

Exercise 3. Sketch the derivatives of graphs 3 and 4.

Graph 3



Graph 4

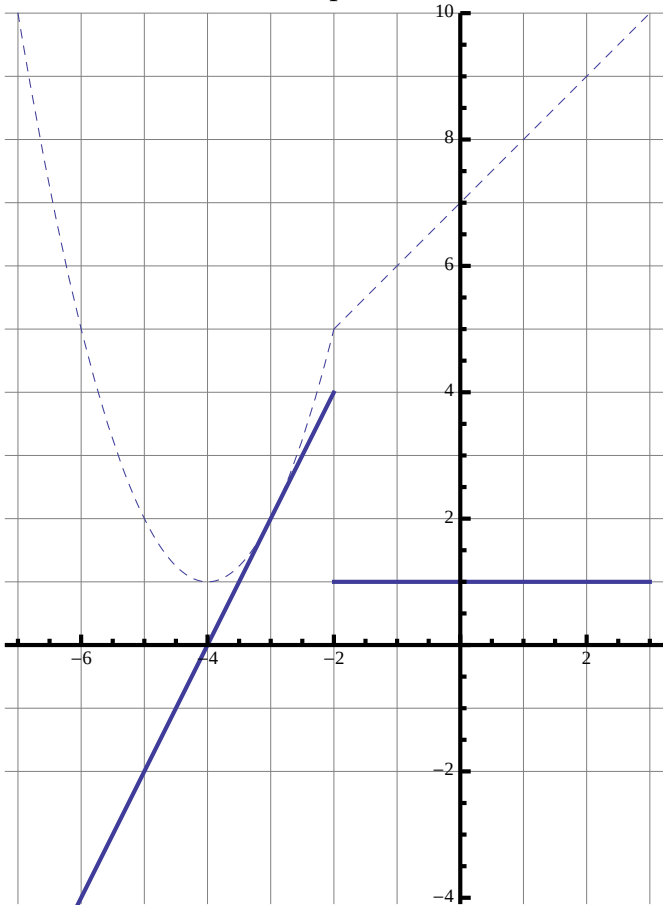


Exercise 4. What is an important difference between the derivative of graph 3 and the derivative of graph 4? Use terminology from calculus.

The derivative of Graph 4 is not continuous.

Exercise 5. Sketch the derivatives of graphs 5 and 6.

Graph 5



Graph 6