§3.1 (Introduction) Some Differentiation Rules

You tell me:

If $f(x) = x^5$, then $f'(x) = \boxed{5x^4}$

Rule you are using? If $f(x) = x^n$, then $f'(x) = \boxed{nx^{n-1}}$

Notation: $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$.

A pretty useful rule: $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$, $f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-2/3}$

$f(x) = x$, $f'(x) = 1$

Why does this rule work?

Let $f(x) = x^n$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{x^n - a^n}{x - a}$$

get $\frac{0}{0}$ when plug in ... so MUST factor!!

$$= \lim_{x \to a} \frac{(x-a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \dots + a^{n-2}x + a^{n-1})}{x-a}$$

$$= \lim_{x \to a} x^{n-1} + ax^{n-2} + a^2x^{n-3} + \dots + a^{n-2}x + a^{n-1}$$

Now plug in!

$$= \lim_{x \to a} \underbrace{a^{n-1} + a^{n-1} + a^{n-1} + \dots + a^{n-1}}_{n \text{ terms}} = na^{n-1}$$

You tell me: $f(x) = 16x^{10}$, $f'(x) = \boxed{16 \cdot 10 \cdot x^9} = 160x^9$

Rule: $\frac{d}{dx}\left[cf(x)\right] = c \cdot f'(x)$ "constants go along for the ride"

Why? $G(x) = cf(x)$

$$G'(x) = \lim_{h \to 0} \frac{G(x+h) - G(x)}{h} = \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h}$$

$$= c\left(\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\right) = c \cdot f'(x)$$

You tell me:

Last topic: If $f(x)=e^x$, then $f'(x)=\boxed{e^x}$

Why? $f(x)=e^x$

$$f'(x)=\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}=e^x\left[\lim_{h\to 0}\left(\frac{e^h-1}{h}\right)\right]=e^x\cdot 1=e^x$$

def: e is defined as the number with the property that: $\lim_{h\to 0}\frac{e^h-1}{h}=1$

graphically, $y=e^x$ has a slope of 1 where $x=0$.