1. Pull out a calculator and complete the charts below:

   (a) The variable θ is in **degrees**.

θ	0	0.001	0.01	0.1
$\frac{\sin(\theta)}{\theta}$	DNE	0.0174533	0.0174533	0.0174533

   (b) The variable θ is in **radians**.

θ	0	0.001	0.01	0.1
$\frac{\sin(\theta)}{\theta}$	DNE	0.999999	0.999983	0.998334

   (c) The variable θ is in **radians**.

θ	0	0.001	0.01	0.1
$\frac{1-\cos(\theta)}{\theta}$	DNE	0.0005	0.00499996	0.0499583

2. Based on the tables above, what would you conclude about:

   * (a) $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1$ if θ in radians (In degrees, it's equal to $\frac{\pi}{180}$)

   * (b) $\lim_{\theta\to 0}\frac{1-\cos(\theta)}{\theta} = 0$

3. Use the definition of the derivative to find the derivative of $y = \sin(x)$ assuming x is measured in radians.

$$y' = \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}$$

use: $\sin(a+b) = \sin a\cos b + \sin b\cos a$

$$= \lim_{h\to 0}\frac{\sin x\cosh + \sinh\cos x - \sin x}{h}$$

re-arrange numerator. Split across denominator

$$= \lim_{h\to 0}\frac{\sin x\cosh - \sin x}{h} + \frac{\cos x\sinh}{h}$$

If no h's, it's a constant!

$$= (\sin x)\left(\lim_{h\to 0}\frac{\cosh - 1}{h}\right) + (\cos x)\left(\lim_{h\to 0}\frac{\sinh}{h}\right)$$

$$= (\sin x)\cdot 0 + (\cos x)(1) = \underline{\underline{\cos x}}$$

*

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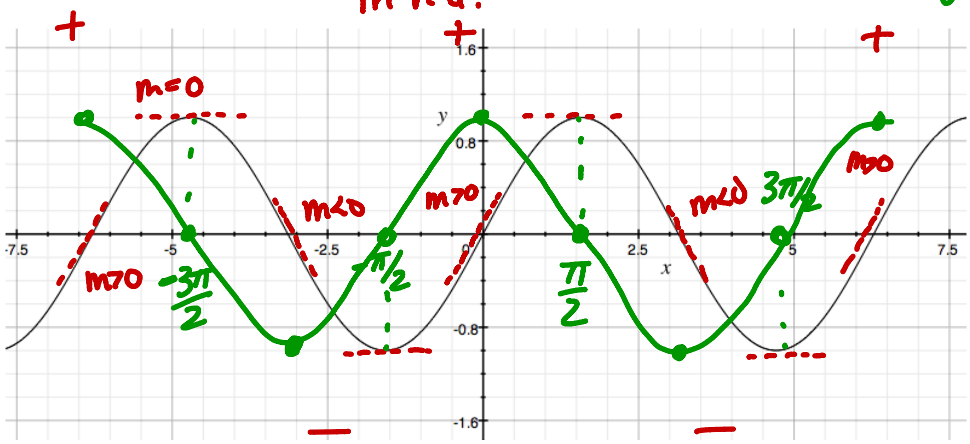
Summary

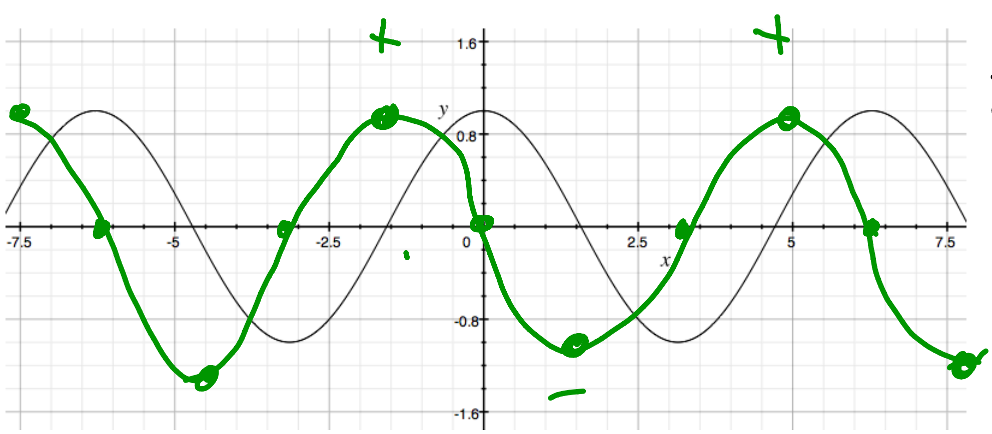
$$\frac{d}{dx}[\sin x] = \cos x$$

4. Use the graph of $y = \sin x$ to sketch a graph of y'. Does this fit with our calculation on the previous page? Why? I will put tangents in red. I will sketch y' in green



Yes! It does fit. The green graph looks + behaves like $y = \cos x$

5. Use the graph of $y = \cos x$ to sketch a graph of y'. What would you guess y' to be and why?



$\frac{d}{dx}[\cos x] = -\sin x.$

$y = \cos x = \sin(x + \frac{\pi}{2})$

6. Use what we learned in 4. and 5. above to find the derivative of:

(a) $y = 3x^4 \cos(x)$

$y' = 12x^3 \cos x - 3x^4 \sin x = 3x^3(4\cos x - x\sin x)$

(b) $y = \quad \csc(x)$ (Use the Quotient Rule.)

$y = \frac{1}{\sin x}$

$y' = \frac{(\sin x)\cdot 0 - 1\cdot\cos x}{(\sin x)^2} = \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x} = -(\cot(x))(\csc(x))$

$= -\csc(x)\cdot\cot(x)$