1. For each function $H(x)$ below, write it as a (nontrivial) composition of functions in the form $f(g(x))$.

   (a) $H(x) = \tan(2 - x^4)$

   $f(x) = \tan x$
   $g(x) = 2 - x^4$

   (b) $H(x) = e^{2-2x}$

   $f(x) = e^x$

   $g(x) = 2 - 2x$

2. Complete the Chain Rule (using both types of notation)

   - If $F(x) = f(g(x))$,

     then $F'(x) = [f'(g(x))][g'(x)]$

   - If $y = f(u)$ and $u = g(x)$,

     then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

3. Find the derivative of the function. You do not need to simplify your answer.

   (a) $y = \sqrt[3]{4 - 2x} = (4-2x)^{1/3}$ $f = x^{1/3}$ $g = 4-2x$

   $y' = \frac{1}{3}(4-2x)^{-2/3}(-2) = -\frac{2}{3}(4-2x)^{-2/3}$

   (b) $f(x) = \underset{f}{0.04\sin}(\underset{g}{3x + e^x})$

   $f'(x) = (0.04)(\cos(3x+e^x))(3+e^x)$

(c) $x(t) = \dfrac{e^{-\pi t^2/10}}{100}$ (**Don't** use the quotient rule here!)

$$x(t) = \frac{1}{100}\, e^{\left(-\frac{\pi}{10}\right)t^2}$$

$$x'(t) = \frac{1}{100}\left(e^{-\frac{\pi}{10}t^2}\right)\left(-\frac{\pi}{10}\cdot 2\right) = \frac{-\pi}{500}\, e^{-\frac{\pi}{10}t^2}$$

(d) $g(x) = \dfrac{50\sqrt{2}}{x + \tan x}$ (**Don't** use the quotient rule here!)

$$g(x) = 50\sqrt{2}\,(x+\tan x)^{-1}$$

$$g'(x) = \left(50\sqrt{2}\right)(-1)(x+\tan x)^{-2}\left(1+\sec^2 x\right)$$

4. Suppose that $f(x) = x^3,\ g(x) = \cos(x)$ and $h(x) = 7 + e^x$.

(a) Find $F(x) = f(x)\left(g(h(x))\right)$, then find its derivative.

$$F(x) = x^3\cos(7+e^x)$$

$$F'(x) = 3x^2\cos(7+e^x) + x^3\left(-\sin(7+e^x)\right)\left(e^x\right)$$

$$= 3x^2\cos(7+e^x) - x^3e^x\sin(7+e^x)$$

(b) Find $G(x) = f(\ g(x)h(x)\ )$, then find its derivative.

$$g(x)\,h(x) = (\cos x)(7+e^x) = 7\cos x + e^x\cos x$$

$$G(x) = (7\cos x + e^x\cos x)^3$$

$$G'(x) = 3(7\cos x + e^x\cos x)^2\left(-7\sin x + e^x\cos x - e^x\sin x\right)$$

$$= 3\left(e^x\cos x - (7+e^x)\sin x\right)(7\cos x + e^x\cos x)^2$$

$h(x) = 7 + e^x$

(c) Find $K(x) = \dfrac{g(x)}{h(f(x))}$, then find its derivative.

$h(f(x)) = 7 + e^{x^3}$

So $K(x) = \dfrac{\cos x}{7 + e^{x^3}}$

So $K'(x) = \dfrac{(7 + e^{x^3})(-\sin x) - (\cos x)(e^{x^3}(3x^2))}{(7 + e^{x^3})^2}$ ✓

(d) Find $G(x) = f(g(h(x)))$, then find its derivative.

$G(x) = (\cos(7 + e^x))^3$

$G'(x) = 3(\cos(7 + e^x))^2(-\sin(7 + e^x))(e^x)$

$= -3e^x \sin(7 + e^x)[\cos(7 + e^x)]^2$