1. Find $\frac{dy}{dx}$ for each of expression below by implicit differentiation.

(a) $2x + 3y = 5xy + y^{1/3}$

$$2 + 3y' = 5 \cdot y + 5xy' + \frac{1}{3} y^{-2/3} \cdot y'$$

$$2 - 5y = 3y' + 5xy' + \frac{1}{3} y^{-2/3} \cdot y' = \left(3 + 5x + \frac{1}{3} y^{-2/3}\right) y'$$

So $$\frac{dy}{dx} = \frac{2 - 5y}{3 + 5x + \frac{1}{3} y^{-2/3}} \cdot \frac{3y^{2/3}}{3y^{2/3}} = \frac{3y^{2/3}(2 - 5y)}{9y^{2/3} + 15xy^{2/3} + 1}$$

(b) $y \sin(x) = x^2 - y^2$

$$y' \cdot \sin(x) + \boxed{y \cos x} = 2x - \boxed{2yy'}$$

$$y' \sin x + 2yy' = 2x - y \cos x$$

$$(\sin x + 2y)\, y' = 2x - y \cos x$$

$$y' = \frac{2x - y \cos x}{\sin x + 2y}$$

(c) $e^{xy} = x + y + 1$

$$e^{xy} \cdot [1 \cdot y + x \cdot y'] = 1 + y'$$

$$\boxed{ye^{xy}} + xe^{xy} y' = 1 + \boxed{y'}$$

$$xe^{xy} y' - y' = 1 - ye^{xy}$$

$$(xe^{xy} - 1)\, y' = 1 + ye^{xy}$$

$$y' = \frac{1 + ye^{xy}}{xe^{xy} - 1}$$

2. You are going to derive the formula for the derivative of arc tangent the way we derived the derivative for arc sine at the beginning of class.

   (a) Find dy/dx for the expression $x = \boxed{\tan(y)}$.

$$1 = (\sec^2 y) \cdot y'$$

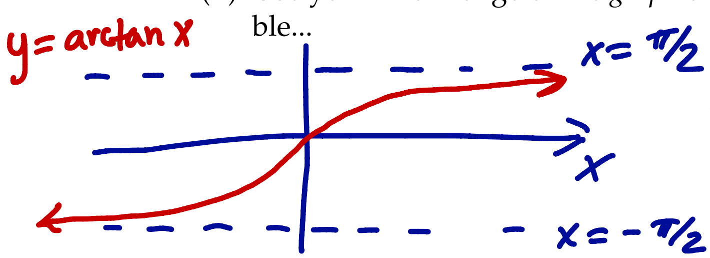
$$y' = \frac{1}{\sec^2 y}$$

   (b) Use the identity $1 + \tan^2(\theta) = \sec^2(\theta)$ to rewrite you answer in part (a) and *write your dy/dx in terms of x only*.

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \boxed{\tan^2 y}} = \frac{1}{1 + x^2}$$

   (c) Now fill in the blank $\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$

   (d) Use your knowledge of the *graph* of $f(x) = \arctan(x)$ to decide if your answer seems plausible...



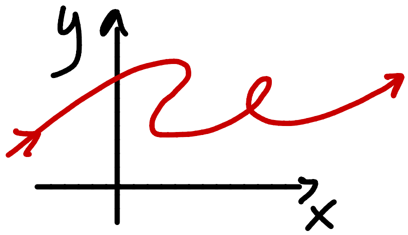
observe: All tangents have POSITIVE slope. AND all values of $\frac{1}{1+x^2}$ are positive!!

3. Find the derivative of $f(x) = \arctan(\sqrt{4 - x^2})$.

$$f'(x) = \left(\frac{1}{1 + (\sqrt{4-x^2})^2}\right)\left(\frac{1}{2}(4-x^2)^{-1/2}\right)(-2x)$$

§ 3.5 Implict Differentiation.

① Why? • The path of an ant on a sheet of paper may not form y as a function of x...



• or the familiar: $x^2 + y^2 = 10$

② Solution: Treat y as $f(x)$ and use the Chain Rule

Example: $4x^2 - \boxed{y^2} = 5$ ← fyi hyperbola

*

take derivative

$$8x^2 - 2\cdot y\cdot\frac{dy}{dx} = 0$$

algebra. Solve for $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{8x^2}{2y} = 4x^2y$$

$$y^2 = (f(x))^2 \xrightarrow{\frac{dy}{dx}} 2(f(x))^1 \cdot f'(x) = 2y\cdot\frac{dy}{dx}$$

③ More Challenging Example

$$x^2 + y^3 = x\sin(y)$$

$$2x + 3y^2\cdot y' = 1\cdot\sin(y) + x\cos(y)\cdot y'$$

$$3y^2y' - (x\cos y)y' = \sin(y) - 2x$$

$$y' = (\sin(y) - 2x)/(3y^2 - x\cos y)$$

④ Another use: Find $\frac{d}{dx}\left[\arcsin(x)\right] = \boxed{}$

$\boxed{y = \arcsin(x)}$ is the same as $\boxed{x = \sin(y)}$ (provided $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$)

So: take derivative implicitly:

$$1 = \cos(y) \cdot y'$$

$$\frac{dy}{dx} = \frac{1}{\cos(y)}$$

use:

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \pm\sqrt{1 - \sin^2 y}$

Here:

$\cos y = \sqrt{1 - \sin^2 y}$

replace

$$= \frac{1}{\sqrt{1 - \sin^2 y}}$$

replace

$$= \frac{1}{\sqrt{1 - x^2}}$$

Summary: $\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1 - x^2}}$

If $y = \arcsin(5x+1)$, $y' = \frac{1}{\sqrt{1 - (5x+1)^2}}(5) = \frac{5}{\sqrt{1 - (5x+1)^2}}$