§ 3.6

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How to find $\frac{d}{dx}\left[\log_b x\right] = \boxed{\frac{1}{(\ln b)x}}$?

Use implicit differentiation.

$\boxed{y = \log_b x} \iff \boxed{b^y = x}$

two ways of writing the same thing!

Find dy/dx implicitly

$\underbrace{(b^y)(\ln b)}_{\text{derivative of } b^{\square}} \cdot \underbrace{\frac{dy}{dx}}_{\text{derivative of inside } \square \text{ namely } y} = 1$

Solve: $\frac{dy}{dx} = \frac{1}{(\ln b)\, b^y} = \boxed{\frac{1}{(\ln b)x}}$

See box at top.

Use $\ln e = 1$ to get: $\frac{d}{dx}\left[\ln x\right] = \frac{1}{(\ln e)x} = \frac{1}{x}$

1. Fill in the derivative rules below:

$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\arccos(x)] = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$

$\frac{d}{dx}[\log_b(x)] = \frac{1}{(\ln b)x}$

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

2. Find the derivative of each function below:

(a) $y = \ln(x^5) = 5\ln x$

$y' = 5 \cdot \frac{1}{x} = \frac{5}{x}$

(b) $y = (\ln x)^5$

$y' = 5(\ln x)^4\left(\frac{1}{x}\right) = \frac{5(\ln x)^4}{x}$

(c) $f(x) = 9x + 4\arctan(3x) + 3\ln(5x)$

$f'(x) = 9 + 4\left(\frac{1}{1+(3x)^2}(3)\right) + 3\left(\frac{1}{5x}\right)(5)$

$= 9 + \frac{12}{1+9x^2} + \frac{3}{x}$

(d) $f(x) = x\log_2(x)$

$$f'(x) = 1 \cdot \log_2 x + x \cdot \frac{1}{(\ln 2)x} = \log_2 x + \frac{1}{\ln 2}$$

(e) $g(x) = \ln(x^2+1)$

$$g'(x) = \frac{2x}{x^2+1} \leftarrow \text{hey! } \frac{d}{dx}\left[\ln(f(x))\right] = \frac{f'(x)}{f(x)}$$

3. Find $\frac{dy}{dx}$ for $y = \ln\left(\frac{x+\sin x}{x^2-e^x}\right)^{1/2}$. (use log rules!)

$$y = \frac{1}{2}\left[\ln\left(\frac{x+\sin x}{x^2-e^x}\right)\right]$$

$$y = \frac{1}{2}\ln(x+\sin x) + \frac{1}{2}\ln(x^2-e^x)$$

Now take derivative:

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x+\sin x}\right)(1+\cos x) + \frac{1}{2}\left(\frac{1}{x^2-e^x}\right)(2x-e^x)$$

$$= \frac{1+\cos x}{2(x+\sin x)} + \frac{2x-e^x}{2(x^2-e^x)}$$